

2021

MATHEMATICS

PAPER-II

Time Allowed — 3 Hours

Full Marks — 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.

Group-A

Answer any four questions.

1. (a) Show that each root of $x^7 - 1 = 0$ is a non negative integral power of $\alpha = \cos \frac{2k\pi}{7} + i \sin \frac{2k\pi}{7}$, $k \in \mathbb{Z}$ and also prove that the roots of $x^2 + x + 2 = 0$ are $(\alpha + \alpha^2 + \alpha^4)$ and $(\alpha^3 + \alpha^5 + \alpha^6)$. 5+10=15
- (b) Find the special roots of the equation $x^9 - 1 = 0$ and show that they are roots of the equation $x^6 + x^3 + 1 = 0$. 5+5=10
2. (a) Suppose H is the only subgroup of finite order m in the group G. Prove that H is a normal subgroup of G. If G is a finite group and H is a normal subgroup of G, then prove that $O\left(\frac{G}{H}\right) = \frac{O(G)}{O(H)}$. 10+5=15
- (b) Let Z denote the centre of a group G. If G/Z is cyclic group prove that G is abelian. 10
3. (a) Determine the point where the function is minimum $x^2 + y^2 + (x + y + 1)^2$. 10
- (b) (i) If the vectors $\vec{A}$ and $\vec{B}$ be irrotational, then show that the vector $\vec{A} \times \vec{B}$ is solenoidal.
- (ii) Use Stokes' theorem to evaluate $\oint_C (\cos x dx + 2y^2 dy + z dz)$, where C is the curve $x^2 + y^2 = 1, z = 1$. 5+10=15
4. (a) Let l_p be the set of all real sequences for which $\sum_{i=1}^{\infty} |x_i|^p < \infty$. We define the metric d in l_p by $d(x, y) = \left(\sum_{i=1}^{\infty} |x_i - y_i|^p \right)^{\frac{1}{p}}$, $\forall x = \{x_i\}$ and $y = \{y_i\} \in l_p$, then prove that the space (l_p, d) is a complete metric space. 15

- (b) Consider the function f defined by

$$f(z) = 0, \text{ when } z = 0$$

$$= \frac{x^3 - y^3}{x^2 + y^2} + i \frac{(x^3 + y^3)}{x^2 + y^2}, \text{ when } z \neq 0.$$

Then show that the function f satisfies the Cauchy-Riemann equation at the origin. 10

5. (a) Prove that $\iint \{2a^2 - 2a(x+y) - (x^2 + y^2)\} dx dy = 8\pi dt$. The region of integration being the circle $x^2 + y^2 + 2a(x+y) = 2a^2$. 15
- (b) Show that $1! 3! 5! \dots (2n-1)! > (n!)^n$. 10

Group-B

Answer any four questions.

6. (a) Use Picard's method to compute $y(0.1)$, from the differential equation $\frac{dy}{dx} = x + y$; $y = 1$, when $x = 0$, correct to five decimal places. 15
- (b) Express the Boolean function $f = x + (x' \cdot y' + x' \cdot z')$ in full disjunctive normal form. 10
7. (a) Find the mean and standard deviation of the following probability distribution
 $f(x) = Ce^{-\frac{1}{24}(x^2 - 6x + 5)}$, $-\infty < x < \infty$. Find C. $2\frac{1}{2} + 2\frac{1}{2} + 10 = 15$
- (b) If independent random variables X and Y be each uniformly distributed in the interval $(-a, a)$, then find the distribution of
- (i) $X + Y$
- (ii) $\frac{X}{Y}$ $5 + 5 = 10$
8. (a) The first two moments of a sample of size ' n ' about the value 4 are 12 and 220 respectively. Find the mean and variance of the observation. $5 + 10 = 15$
- (b) If the relation between variables X and Y , U and V are $2X + 3Y = 4$, $3U + 4V = 5$ and the regression coefficient of X on U is 4, find the regression coefficient of Y on V . 10

9. (a) Write down the dual of the following L.P.P.

$$\text{Maximize } Z = 3x_1 + 4x_2$$

$$\text{Subject to } x_1 + x_2 \leq 10$$

$$2x_1 + 3x_2 \leq 18$$

$$x_1 \leq 8$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0.$$

Also solve it by using dual problem.

5+10=15

- (b) Find the optimal solution and the corresponding cost of transportation in the following transportation problem: 10

	D ₁	D ₂	D ₃	D ₄	a _i
O ₁	19	20	50	10	7
O ₂	70	30	40	60	9
O ₃	40	8	70	20	18
b _j	5	8	7	14	

10. (a) Draw a circuit which realises the Boolean function $f = (x + y).(y + z).(z + x)$. Use the law of Boolean algebra to show that the above circuit is equivalent to a switching circuit in which if any two switches are on, the light is on and construct the equivalent switching circuit.

5+10=15

- (b) There are 40 students in Statistics (Hons) class in a particular college. If the habit of borrowing books from college library of a student is $\frac{2}{5}$, find the minimum number of copies of a book, referred in a class, to be kept in the library so to meet more than 90% demand of the students.

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