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CSM—47/24

PART—II/PAPER—VII

MATHEMATICS

PAPER—II

Time : 3 Hours

Full Marks : 250

The question paper contains **18 (Eighteen)** *questions in* **GROUP—A, (12)** *and* **GROUP—B, (06)** *together.*

GROUP—A

Candidates to attempt **10 *(ten)*** *questions.*

Each question carries **15** *marks.*

1. Find a complete integral of

$$p^2x + q^2y = z$$

by Charpit's method, where

$$p \equiv \frac{\partial z}{\partial x} \text{ and } q \equiv \frac{\partial z}{\partial y}$$

2. A particle of mass m is projected into the air with velocity u in a direction making an angle α with the horizontal. Find its motion and the path described.

3. Write a FORTRAN program to find the roots of a quadratic equation of type $ax^2 + bx + c = 0$ provided $a \neq 0$.

4. Using Laplace transform, find the solution of

$$\frac{d^2y}{dt^2} + y = 8\cos t$$

given that $y(0) = -1$, $y'(0) = 1$.

5. Find the root of the equation

$$x^3 - x - 4 = 0$$

between 1 and 2 by the method of bisection, correct to 3 decimal places.

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6. Find a solution of the differential equation

$$\frac{dy}{dx} - y\tan x = 0$$

in the form $y = y_1(x)$, say. Hence solve

$$\frac{dy}{dx} - y\tan x = \cos x$$

by the substitution $y = y_1(x)\,v(x)$.

7. Solve graphically the following L.P.P :

Maximize $Z = 6x_1 + 10x_2$

subject to $3x_1 + 5x_2 \le 10$

$5x_1 + 3x_2 \le 15$

$x_1, x_2 \ge 0$

8. Calculate the positive root of the equation

$$x^2 + 2x - 2 = 0$$

correct up to two significant figures by the Newton-Raphson method.

9. Solve the transportation problem with the following unit cost matrix by obtaining the initial solution using matrix minima method :

	D_1	D_2	D_3	D_4	a_i
O_1	8	10	7	6	50
O_2	12	9	4	7	40
O_3	9	11	10	8	30
b_j	25	32	40	23	120

10. The values of $y = \sin x$ are given below for different values of x. Form the difference table. Also find the value of $\sin 32°$ from this table.

x	30°	35°	40°	45°	50°	55°
$\sin x$	0·5000	0·5736	0·6428	0·7071	0·7660	0·8192

11. Three forces P, Q, R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$, $y = 2$. Find the equation of the line of action of their resultant.

12. Show that any connected graph with n vertices and $n-1$ edges is a tree.

GROUP—B

Candidates to attempt **05** ***(five)*** *questions.*

Each question carries **20** *marks.*

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13. Find the general power series solution near $x = 0$ of the Legendre differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

where n is an arbitrary constant.

14. Write an integer FUNCTION subprogram to find the factorial of a positive integer k.

15. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}$ rule taking $n = 6$. Hence compute an approximate value of π, correct to 5 significant figures.

16. A rough surface is formed by the revolution of a rectangular hyperbola about a vertical asymptote. Show that a particle will rest on it anywhere beyond its intersection with a certain circular cylinder.

17. Solve the following initial value problem using Euler's method :

$$\frac{dy}{dx} = x^2 + y, \; y(0) = 0$$

Compute the first steps of the solution with $h = 0{\cdot}1$. Compare the result (% relative error) with those obtained from the exact solution $y = 3e^x - x^2 - 2x - 2$.

18. Solve the following L.P.P. by simplex method :

Maximize $Z = 6x_1 - 2x_2$

subject to $2x_1 - x_2 \leq 2$

$x_1 \leq 4$

$x_1, x_2 \geq 0$

★★★

SPACE FOR ROUGH WORK

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