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CSM—46/24

PART—II/PAPER—VI

MATHEMATICS

PAPER—I

Time : 3 Hours

Full Marks : 250

The question paper contains **18 (Eighteen)** *questions in* **GROUP—A, (12)** *and* **GROUP—B, (06)** *together.*

GROUP—A

Candidates to attempt **10 *(ten)*** *questions.*

Each question carries **15** *marks.*

1. Let W_1 and W_2 be any two subspaces of a finite dimensional vector space $V(F)$, then prove that

$$\dim V = \dim W_1 + \dim W_2$$

2. Let $V(F)$ be a finite dimensional vector space. Then show that distinct non-zero eigenvectors of a linear transformation $T: V \to V$ corresponding to distinct eigenvalues of T are linearly independent.

3. If a and b are two arbitrary elements of an abelian group G, then show that $(ab)^n = a^n b^n$, for all integer n.

4. State and prove Cayley's theorem.

5. State and prove Rank-nullity theorem.

6. Prove that an ideal S of the ring Z of all integers is a maximal ideal if and only if S is generated by some prime integer.

7. Let N be a normal subgroup of a group G and H be any subgroup of G, then prove that

$$\frac{(HN)}{N} \cong \frac{H}{(H \cap N)}$$

8. Find the dimension of the subspace of $\mathbb{R}^4$ spanned by the set $\{(1\ 0\ 0\ 0), (0\ 1\ 0\ 0), (1\ 2\ 0\ 1), (0\ 0\ 0\ 1)\}$. Hence find its basis.

9. Prove that the general equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents two parallel straight lines if $h^2 = ab$ and $bg^2 = af^2$. Also prove that the distance between them is

$$\frac{2\sqrt{g^2 - ac}}{\sqrt{a(a+b)}}$$

10. Find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}, \quad \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

Find also its equations and the points in which it meets the given lines.

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that $A^n = A^{n-2} + A^2 - I$, for $n \geq 3$.

Hence, find A^{50}.

12. Find the equation of a sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point $(1, -2, 1)$ and also cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.

GROUP—B

Candidates to attempt **05 *(five)*** *questions.*

Each question carries **20** *marks.*

13. Test the convergence of the series

$$\sum \frac{x^{n-1}}{1+x^n}, \quad x > 0$$

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14. Let D be the region bounded by the closed cylinder $x^2 + y^2 = 16$, $z = 0$ and $z = 4$. Verify the divergence theorem if $v = 3x^2 i + 6y^2 j + zk$.

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15. Show that the function $f(z) = u + iv$, where

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$$

$(z \neq 0)$, $f(0) = 0$ is continuous and that the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

16. The function $f(z)$ has a double pole at $z = 0$ with residue 2, a simple pole at $z = 1$ with residue 2, is analytic at all other finite points of the plane and is bounded as $|z| \to \infty$. If $f(2) = 5$ and $f(-1) = 2$, then find $f(z)$.

17. Show that the series

$$\sum_{n=1}^{\infty} \frac{a_n x^n}{1 + x^{2n}}$$

is uniformly convergent for all real x, if $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

18. Examine the continuity of the function f defined by

$$f(x) = x - [x], \quad x \in \mathbb{R}$$

where $[x]$ denotes the greatest integer function.

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SPACE FOR ROUGH WORK

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