CSM—47/23

PART—II/PAPER—VII

MATHEMATICS

ଗଣିତ

PAPER—II

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Time : 3 Hours

ସମୟ : ୩ ଘଣ୍ଟା

Full Marks : 250

ପୂର୍ଣ୍ଣ ସଂଖ୍ୟା : ୨୫୦

The question paper contains **18 (Eighteen)** *questions in* **GROUP—A, (12)** *and* **GROUP—B, (06)** *together.*

ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନପତ୍ରରେ **GROUP—A, (12)** ଏବଂ **GROUP—B, (06)**କୁ ମିଶାଇ **18**ଟି ପ୍ରଶ୍ନ ରହିଛି।

GROUP—A

Candidates to attempt **10** ***(ten)*** *questions.*
Each question carries **15** *marks.*

ପରୀକ୍ଷାର୍ଥୀମାନେ **୧୦**ଟି ପ୍ରଶ୍ନ **୨୫୦** ଶବ୍ଦ ମଧ୍ୟରେ ଉତ୍ତର ଦେବେ।
ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନର ମୂଲ୍ୟ **୧୫** ମାର୍କ।

1. Find the root of the equation $\sin x = 1 + x^3$, between -2 and -1 correct to 3 decimal places by Newton Raphson method.

ସମାଧାନ କର।

Find the root of the equation $\sin x = 1 + x^3$, between -2 and -1 correct to 3 decimal places by Newton Raphson method.

2. Find the value of $e^{1.85}$, given $e^{1.7} = 5.4739$, $e^{1.8} = 6.0496$, $e^{1.9} = 6.6859$, $e^{2.0} = 7.3891$, $e^{2.1} = 8.1662$, $e^{2.2} = 9.0250$, $e^{2.3} = 9.9742$.

ମୂଲ୍ୟ ନିରୂପଣ କର।

Find the value of $e^{1.85}$, given $e^{1.7} = 5.4739$, $e^{1.8} = 6.0496$, $e^{1.9} = 6.6859$, $e^{2.0} = 7.3891$, $e^{2.1} = 8.1662$, $e^{2.2} = 9.0250$, $e^{2.3} = 9.9742$.

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3. Compute by Simpson's one-third rule, the integral $\int_0^1 x^2(1-x)\,dx$ correct to three places of decimal, taking step length equal to 0.1.

ଗଣନ କର।

Compute by Simpson's one-third rule, the integral $\int_0^1 x^2(1-x)\,dx$ correct to three places of decimal, taking step length equal to 0.1.

4. A company has five jobs V, W, X, Y and Z and five machines A, B, C, D and E. The given matrix shows the return in rupees of assigning a job to a machine. Assign the jobs to machines so as to maximize the total returns.

Jobs	A	B	C	D	E
V	5	11	10	2	4
W	2	4	6	3	5
X	3	12	5	14	6
Y	6	14	4	11	7
Z	7	9	8	12	5

ଗୋଟିଏ କମ୍ପାନୀର ପାଞ୍ଚଟି ଚାକିରି ଯଥା ଏବଂ ପାଞ୍ଚଟି ମେସିନ୍ ଯତା ଅଛି। ନିମ୍ନ ପ୍ରଦତ୍ତ ମାଟ୍ରିକ୍ସ ମେସିନକୁ କାମ ଅନୁଯାୟୀ ଟଙ୍କାର ରୋଜଗାର ପିରମାଣ ସୂଚିତ କରୁଛି। ମେସିନ ଅନୁସାରେ କାମ ଏପରି ସଂଯୁକ୍ତ କର। ଯାହା ଦ୍ୱାରା ଅଧିକ ଟଙ୍କା ରୋଜଗାର ହୋଇପାରିବ।

Jobs	A	B	C	D	E
V	5	11	10	2	4
W	2	4	6	3	5
X	3	12	5	14	6
Y	6	14	4	11	7
Z	7	9	8	12	5

5. Given $y' = 1 + xy$ with the initial condition that $y = 1, x = 0$. Compute $y(0.1)$ correct to 4 places of decimal by using Taylor's series method.

ସମାଧାନ କର।

Given $y' = 1 + xy$ with the initial condition that $y = 1, x = 0$. Compute $y(0.1)$ correct to 4 places of decimal by using Taylor's series method.

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6. Prove that a simple graph with n vertices must be connected if it has more than $\frac{1}{2}(n-1)(n-2)$ edges.

ପ୍ରମାଣ କର।

Prove that a simple graph with n vertices must be connected if it has more than $\frac{1}{2}(n-1)(n-2)$ edges.

7. Test for exactness and solve $(1+x^2)y'' + 4xy' + 2y = \sec^2 x$, given that $y = 0, y' = 1$ when $x = 0$.

ପରୀକ୍ଷା କର।

Test for exactness and solve $(1+x^2)y'' + 4xy' + 2y = \sec^2 x$, given that $y = 0, y' = 1$ when $x = 0$.

8. Solve the PDE $(y - xz)p + (x + yz)q = x^2 + y^2$.

ସମାଧାନ କର।

Solve the PDE $(y - xz)p + (x + yz)q = x^2 + y^2$.

9. A company produces soft drinks and has a contract requiring that a minimum of 80 units of chemical A and 60 units of chemical B go into each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers. The supplier X_1 has a mix of 4 units of A and 2 units of B that costs rupees 10 and the supplier X_2 has a mix of 1 unit of A and 1 unit of B that costs rupees 4. How many mixes from the company X_1 and company X_2 should the company purchase to honour contract requirement and yet minimise cost? (Formulate the problem and solve it by graphical method).

କୋଟିଏ କମ୍ପାନୀ ମୃଦୁ ପାନୀୟ ଉତ୍ପାଦନ କରେ ଏବଂ ଏକ ଚୁକ୍ତିନାମା ଅଛି ଯାହା ଅନୁଯାୟୀ ପାନୀୟର ପ୍ରତ୍ୟେକ ବୋତଲରେ ଅତି କପରେ 80 ୟୁନିଟ୍ କେମିକାଲ୍‌ର A ଏବଂ 60 ୟୁନିଟ୍ କୋମିକାଲ୍‌ର B ଦେବାକୁ ପଡ଼ିବ। କେମିକାଲ ଦ୍ୱୟ ଦୁଇଟି ଭିନ୍ନ ଯୋଗାଣକାରୀଙ୍କଠାରୁ ପ୍ରସ୍ତୁତ ମିଶ୍ରଣରେ ଉପଲବ୍ଧ ହୋଇପାଏ। ଯୋଗାଣକାରୀ X_1 ରେ 4 ୟୁନିଟ୍ A ଏବଂ 2 ୟୁନିଟ୍ Bର ମିଶ୍ରଣ ଅଛି ଏହାର-ମୂଲ୍ୟ 10 ଟଙ୍କା। ଯୋଗଣ କାରୀ X_2 ରେ 1 ୟୁନିଟ୍ A ଓ 2 ୟୁନିଟ୍ B ମିଶ୍ରଣ ଅଛି ଏହାର ମୂଲ୍ୟ 4 ଟଙ୍କା। କମ୍ପାନୀ X_1 ଓ କମ୍ପାନୀ X_2 କେତେ ମିଶ୍ରଣ କ୍ରୟ କଲେ କମ୍ପନୀ ଚୁକ୍ତିନାମାର ଆବଶ୍ୟକତା ହ୍ରାସ୍ୟ କରିବ? ଏହ ସହିତି ଖର୍ଚ୍ଚ କମ୍ କରିବ। (ସମସ୍ୟା ପ୍ରସ୍ତୁତ କର ଓ ଗ୍ରାଫିକାଲ ପଦ୍ଧତିରେ ଏହା ସମାଧାନ କର।)

10. Write a FORTRAN program that calculates the real roots of any quadratic equation $ax^2 + bx + c = 0$ for given values of a, b and c. The program should print a message on screen if the roots are imaginary and should also be able to solve the equation if $a = 0$.

ଏକ FORTRAN program ଲେଖ। ଯାହା ଯେ କିବାରି କ୍ୱାଡ୍ରିଟିକ୍ ଇକ୍ୱେସନର $ax^2 + bx + c = 0$ ସହ a, b ଏବଂ cର ପ୍ରଦତ୍ତ ମୂଲ୍ୟ ଗଣନା କର। ଯଦି ରୁଟ୍‌ଗୁଡ଼ିକ କାଳ୍ପନିକ ହୋଇଥାଏ, ତେବେ ପ୍ରୋଗ୍ରାମ ସ୍କ୍ରିନରେ ଏକ ବାର୍ତ୍ତା ପ୍ରିନ୍ଟ କରିବା ଉଚିତ। ଏହି ସହିତ $a = 0$ ସମୀକରଣକୁ ସମାଧାନ କରିବାକୁ ମଧ୍ୟମ ହେବ।

11. Consider a ball of mass m. It is attached to a spring of length r and spring constant k. The spring when uncoiled has length l. The other end of the spring is attached to a body like a pendulum. Derive its equation of motion.

ଗୋଟିଏ ବଲ୍‌ର ମାସ୍ m ବିଚାର କର। ଏହା r ଲମ୍ବ ଏବଂ ସ୍ପ୍ରିଙ୍ଗ କନଷ୍ଟାଣ୍ଟ kର ଏକ ସ୍ପ୍ରିଙ୍ଗ ସହିତ ସଂଯୁକ୍ତ। ଯେତେବେଳେ ସ୍ପ୍ରିଙ୍ଗ uncoiled ଥାଏ ତାର ଲମ୍ବ l ଥାଏ। ସ୍ପ୍ରିଙ୍ଗର ଅନ୍ୟ ପ୍ରନ୍ତ ଏକ ପେଣ୍ଡୁଲମ୍ ପରି ବଡ଼ି ସହିତ ସଂଲଗ୍ନ ହେଇପାଏ। ଏହାର ଗତି ସମୀକରଣ ବାହାର କର।

12. Write a C program that checks if a positive integer is divisible by either 11 or 13, or both. If the integer is a multiple of 11, then the program will return true. Similarly, if the integer is a multiple of 13, then also the program will return true. If the integer is not a multiple of 11 or 13, then the program will return false. Write appropriate user defined functions to perform all these operations.

ଗୋଟିଏ C ପ୍ରୋଗ୍ରାମ ଲେଖ ଯାହା ଏକ positive integer 11 କିମ୍ବା 13 କିମ୍ବା ଉଭୟ ଦ୍ୱାରା ବିଭାଜ୍ୟ କି ନୁହେଁ ଯାଞ୍ଚ କରିବ। ଯଦି 11 ର ଗୁଣିତକ ହୁଏ, ତେବେ ପ୍ରୋଗ୍ରାମ ରିଟର୍ଣ୍ଣ ସତ ହେବ। ସେହିପରି ଯଦି 13 ର ଗୁଣିତକ ହୁଏ, ତେବେ ମଧ୍ୟ ପ୍ରୋଗ୍ରାମ ରିଟର୍ଣ୍ଣ ସତ ହେବ। ଯଦି 11 ଓ 13 ର ଗୁଣିତକ ନୁହେଁ, ତେବେ ପ୍ରୋଗ୍ରାମ ରିଟର୍ଣ୍ଣ ମିଥ୍ୟା ହେବ। ଏସମସ୍ତ ପ୍ରକ୍ରିୟା ପାଇଁ ଉପଯୁକ୍ତ user defined functions ଲେଖ।

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GROUP—B

Candidates to attempt **05** ***(five)*** *questions.*

Each question carries **20** *marks.*

ପରୀକ୍ଷାର୍ଥୀମାନେ ୦୫ଟି ପ୍ରଶ୍ନ ୩୦୦ ଶବ୍ଦ ମଧ୍ୟରେ ଉତ୍ତର ଦେବେ।

ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନର ମୂଲ୍ୟ ୨୦ ମାର୍କ।

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13. Solve the LPP : Min. $Z = x - 3y + 2z$, s.t. $3x + y + 3z \leq 7$, $-2x + 4y \leq 12$, $-4x + 3y + 8z \leq 10$ and $x, y, z \geq 0$.

ପ୍ରମାଣ କର।

Solve the LPP : Min. $Z = x - 3y + 2z$, s.t. $3x + y + 3z \leq 7$, $-2x + 4y \leq 12$, $-4x + 3y + 8z \leq 10$ and $x, y, z \geq 0$.

14. Find $y(4.4)$, by Euler's modified method taking $h = 0.2$ from the differential equation $\frac{dy}{dx} = \frac{2 - y^2}{5x}$, $y = 1$ when $x = 4$.

ସମାଧାନ କର।

Find $y(4.4)$, by Euler's modified method taking $h = 0.2$ from the differential equation $\frac{dy}{dx} = \frac{2 - y^2}{5x}$, $y = 1$ when $x = 4$.

15. A company has three factories X, Y and Z and four warehouses A, B, C and D. It is required to schedule factory production and shipments from factories to warehouses in such a manner so as to minimize total cost of shipment and production. Unit variable manufacturing costs (UVMC) and factory capacities and warehouse requirements are given below :

Find the optimal production and transportation schedule.

From	UVMC	To warehouse				Capacity in
Factories	₹	Unit shipping costs in ₹				Units per month
		A	*B*	*C*	*D*	
X	10	0	1	1	2	75
Y	11	1	2	3	1	32
Z	12	4	3	3	6	67
Requirement		65	24	16	15	

ଗୋଟିଏ କମ୍ପାନୀର X, Y ଏବଂ Z ନାମରେ ତିନୋଟି factory ଓ A, B, C ଏବଂ D ନାମରେ ଚାରୋଟି ଗୋଦାମଘର ଅଛି। Factory ଓ ଗୋଦାମଘରକୁ ଉତ୍ପାଦକ ଓ ପରିବହନକୁ ଏପରିଥାଏ ସୂଚୀବଦ୍ଧ କରିବା ଆବଶ୍ୟକ, ଯାହା ଦ୍ୱାରା ପରିବହନ ଓ ଉତ୍ପାଦକ ମୋଟ ଖର୍ଚ୍ଚ କମାଯାଇ ପାରିବ। ୟୁନିଟ୍ ପରିବର୍ତ୍ତନଶୀଳ ଉତ୍ପାଦନ ଖର୍ଚ୍ଚ ଏବଂ କାରଖାନା କ୍ଷମତା ଏବଂ ଗୋଦାମ ଆବଶ୍ୟକକି। ନିମରେ ପ୍ରଦତ୍ତ ହୋଇଛି। ସମ୍ପୋତ୍ତୀ ଉତ୍ପାଦନ ଓ ପରିବହନ ସମୟସୂଚୀ ପ୍ରସ୍ତୁତ କର।

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From	UVMC	To warehouse				Capacity in
Factories	₹	Unit shipping costs in ₹				Units per month
		A	*B*	*C*	*D*	
X	10	0	1	1	2	75
Y	11	1	2	3	1	32
Z	12	4	3	3	6	67
Requirement		65	24	16	15	

16. Let $x = c$ be a root of $f(x) = 0$, which is equivalent to $x = g(x)$ and I be an interval containing c. If $|g'(x)| < 1$ for all x in I, then the sequence of approximations $x_0, x_1, \ldots$ will converge to the root c, provided that the initial approximation x_0 is chosen in I.

ସମାଧାନ କର।

Let $x = c$ be a root of $f(x) = 0$, which is equivalent to $x = g(x)$ and I be an interval containing c. If $|g'(x)| < 1$ for all x in I, then the sequence of approximations $x_0, x_1, \ldots$ will converge to the root c, provided that the initial approximation x_0 is chosen in I.

17. Using Runge-Kutta method of order 4, find y for $x = 0.1, 0.2, 0.3$, given that $\frac{dy}{dx} = xy + y^2, y(0) = 1$.

ପ୍ରମାନ କର।

Using Runge-Kutta method of order 4, find y for $x = 0.1, 0.2, 0.3$, given that $\frac{dy}{dx} = xy + y^2, y(0) = 1$.

18. Determine the flow pattern inside a cylinder which rotates about its vertical axis with constant angular velocity Ω and the surface being open to the atmosphere. (Assume that the atmospheric pressure is constant and the flow is in the gravitational field.)

ପ୍ରମାଣ କର।

Determine the flow pattern inside a cylinder which rotates about its vertical axis with constant angular velocity Ω and the surface being open to the atmosphere. (Assume that the atmospheric pressure is constant and the flow is in the gravitational field.)

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SPACE FOR ROUGH WORK

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