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CSM—46/23

PART—II/PAPER—VI

MATHEMATICS

ଗଣିତ

PAPER—I

Time : 3 Hours

ସମୟ : ୩ ଘଣ୍ଟା

Full Marks : 250

ପୂର୍ଣ୍ଣ ସଂଖ୍ୟା : ୨୫୦

The question paper contains **18 (Eighteen)** *questions in* **GROUP—A, (12)** *and* **GROUP—B, (06)** *together.*

ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନପତ୍ରରେ **GROUP—A, (12)** ଏବଂ **GROUP—B, (06)**କୁ ମିଶାଇ **18**ଟି ପ୍ରଶ୍ନ ରହିଛି।

GROUP—A

ଗ୍ରୁପ୍—A

Candidates to attempt **10 *(ten)*** *questions.*

Each question carries **15** *marks.*

ପରୀକ୍ଷାର୍ଥୀମାନେ **୧୦**ଟି ପ୍ରଶ୍ନ **୨୫୦** ଶବ୍ଦ ମଧ୍ୟରେ ଉତ୍ତର ଦେବେ।

ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନର ମୂଲ୍ୟ **୧୫** ମାର୍କ।

1. Let H, K be subgroups of a group G and K be normal in G. Then prove that $H/(H \cap K)$ is isomorphic to $(HK)/K$.

ମନେକର G group ର subgroup ହେଉଛି H, K ଏବଂ G ରେ K ହେଉଛି normal ତେବେ ପ୍ରମାଣ କରଯେ $H/(H \cap K)$ ହେଉଛି $(HK)/K$ ର ସମରୂପବିଶିଷ୍ଟ।

2. Prove that a finite group G of order n, is isomorphic to a subgroup of symmetric group of order n.

ପ୍ରମାଣକର ଯେ n କ୍ରମର ଏକ ସୀମିତ group G ହେଉଛି n କ୍ରମର ପ୍ରତିସମ group ର subgroup ପାଇଁ ମମରୂପବିଶିଷ୍ଟ।

3. Find maximal ideals and prime ideals in the ring $\mathbb{Z}_6$. Also find the units in the integral domain $\mathbb{Z}[\sqrt{-5}]$ (where $\mathbb{Z}$ represents the set of all integers).

ring $\mathbb{Z}_6$ ରେ ସର୍ବାଧିକ ଆଦର୍ଶ ଏବଂ ଅଧିକ ଗୁରୁତ୍ୱପୂର୍ଣ୍ଣ ଆଦର୍ଶ ନିର୍ଣ୍ଣୟ କର। integral domain $\mathbb{Z}[\sqrt{-5}]$ ରେ ଥିବା ଏକକ ମଧ୍ୟ ନିର୍ଣ୍ଣୟ କର (ଯେଉଁଠାରେ $\mathbb{Z}$ ସମସ୍ତ ସଂଖ୍ୟାର set କୁ ପ୍ରତିନିଧିତ୍ୱ କରେ।)

4. Express the following ideals as a principal ideal in the integral domain $\mathbb{Z}$:

(i) $4\mathbb{Z}+6\mathbb{Z}$

(ii) $8\mathbb{Z}\cap 12\mathbb{Z}$ (where $\mathbb{Z}$ represents the set of all integers).

integral domain $\mathbb{Z}$ ରେ ଥିବା ନିମ୍ନୋକ୍ତ ଆଦର୍ଶକୁ ମୁଖ୍ୟଆଦର୍ଶ ରୂପେ ପ୍ରକାଶ କର।

(i) $4\mathbb{Z}+6\mathbb{Z}$

(ii) $8\mathbb{Z}\cap 12\mathbb{Z}$ (ଯେଉଁଠାରେ $\mathbb{Z}$ ସମସ୍ତସଂଖ୍ୟାର set କୁ ପ୍ରତିନିଧିତ୍ୱ କରେ।)

5. Show that the mapping $\phi:\mathbb{Z}[x]\to\mathbb{Z}$ defined by $\phi(f(x))=f(0)$, for $f(x)\in\mathbb{Z}[x]$ is a homomorphism and find $\ker\phi$. Also deduce that the ideal $<x>$ is the prime ideal in $\mathbb{Z}[x]$, but not a maximal ideal in $\mathbb{Z}[x]$ (where $\mathbb{Z}$ represents the set of all integers).

ଦର୍ଶାଇଦିଅଯେ mapping $\phi:\mathbb{Z}[x]\to\mathbb{Z}$ defined by $\phi(f(x))=f(0)$ ପ୍ରଦତ୍ତ ଯେ $f(x)\in\mathbb{Z}[x]$ ହେଉଛି homomorphism ଏବଂ $\ker\phi$ ନିର୍ଣ୍ଣୟ କର। ଆହୁରି ମଧ୍ୟ ଯୁକ୍ତିସଙ୍ଗତ ଅନୁମାନ କର ଯେ ଆଦର୍ଶ $<x>$ ହେଉଛି $\mathbb{Z}[x]$ ରେ ଅଧିକ ଗୁରୁତ୍ୱପୂର୍ଣ୍ଣ ଆଦର୍ଶ, କିନ୍ତୁ $\mathbb{Z}[x]$ ରେ ଏକ ସର୍ବାଧିକ ଆଦର୍ଶ ନୁହେଁ। (ଯେଉଁଠାରେ $\mathbb{Z}$ ସମସ୍ତ ସଂଖ୍ୟାର set ପ୍ରତିନିଧିତ୍ୱ କରେ।)

6. Let $V=\{(x,y):x,y\in\mathbb{R}\}\equiv\mathbb{R}^2$. The vector addition (+) and scalar multiplication (.) in V is defined as follows :

$$(x_1,y_1)+(x_2,y_2)=(x_1+x_2+1,\, y_1+y_2-2)$$

$$a.\,(x,y)=(ax+a-1,\, ay-2a+2)$$

Then show that V is a vector space over the set of real number $\mathbb{R}$.

ମନେକର $V=\{(x,y):x,y\in\mathbb{R}\}\equiv\mathbb{R}^2$. vector addition (+) ଏବଂ scalar multiplication (.) in V କୁ ନିମ୍ନମତେ ସଂଜ୍ଞା ଦେଇହେବ।

$$(x_1,y_1)+(x_2,y_2)=(x_1+x_2+1,\, y_1+y_2-2)$$

$$a.\,(x,y)=(ax+a-1,\, ay-2a+2)$$

ତେବେ ଦର୍ଶାଅ ଯେ V is a vector space over the set of real number $\mathbb{R}$।

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7. Is the vector $(2,-5,3)$ in the subspace of $\mathbb{R}^3$ which is spanned by the vectors $(1,-3,2)$, $(2,-4,-1)$, $(1,-5,7)$? Also in the vector space $\mathbb{R}^3$, if $\alpha=(1,2,1)$, $\beta=(3,1,5)$, $\gamma=(3,-4,7)$, then show that the subspace spanned by $S=\{\alpha,\beta\}$ and $T=\{\alpha,\beta,\gamma\}$ are the same (where $\mathbb{R}$ represents the set of real numbers).

$\mathbb{R}^3$ subspace ରେ vector $(2,-5,3)$ ଯଦି vectors $(1,-3,2)$, $(2,-4,-1)$, $(1,-5,7)$ ବାଧା ସୃଷ୍ଟି କରୁଥାଏ ଏବଂ $\mathbb{R}^3$ vector space ଯଦି $\alpha=(1,2,1)$, $\beta=(3,1,5)$, $\gamma=(3,-4,7)$, ତେବେ ଦର୍ଶାଅ ଯେ subspace spanned by $S=\{\alpha,\beta\}$ and $T=\{\alpha,\beta,\gamma\}$ ଏକା ଅଟନ୍ତି। (ଯେଉଁଠାରେ $\mathbb{R}$ set of real numbers କୁ ପ୍ରତିନିଧିତ୍ୱ କରେ।)

8. In $V_3(\mathbb{R})$, where $\mathbb{R}$ is the field of real numbers. Examine each of the following sets of vectors for linearly dependents :

(i) $\{(2,3,5),(4,9,25)\}$

(ii) $\{(1,2,1),(3,1,5),(3,-4,7)\}$

(iii) $\{(1,2,2),(2,1,2),(2,2,1)\}$

$V_3(\mathbb{R})$ ରେ (ଯେଉଁଠାରେ $\mathbb{R}$ ହେଉଛି field of real numbers।) Linearly dependent ପାଇଁ ନିମ୍ନଲିଖିତ sets of vectors କୁ ପରୀକ୍ଷା କରି ଦର୍ଶାଅ :

(i) $\{(2,3,5),(4,9,25)\}$

(ii) $\{(1,2,1),(3,1,5),(3,-4,7)\}$

(iii) $\{(1,2,2),(2,1,2),(2,2,1)\}$

9. Let $T:\mathbb{R}^2\to\mathbb{R}^3$ be a linear transformation defined by $T(x,y)=(x-2y,\ 2x+y, x+y)$. Determine the matrix of linear transformation T with respect to ordered bases $\mathbf{B}\mathbb{R}^2=\{(1,-1),(0,1)\}$ and $\mathbf{B}\mathbb{R}^3=\{(1,1,0),(0,1,1),(1,-1,1)\}$, where $\mathbf{B}\mathbb{R}^2$ and $\mathbf{B}\mathbb{R}^3$ are the bases for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively (where $\mathbb{R}$ represents the set of real numbers).

ଯଦି $T(x,y)=(x-2y,\ 2x+y, x+y)$ ଦ୍ୱାରା ସ୍ଥିରୀକୃତ $T:\mathbb{R}^2\to\mathbb{R}^3$ ଏକ linear transformation ହୁଏ, ତେବେ $\mathbf{B}\mathbb{R}^2=\{(1,-1),(0,1)\}$ ଏବଂ $\mathbf{B}\mathbb{R}^3=\{(1,1,0),(0,1,1),(1,-1,1)\}$, ordered bases ତୁଳନାରେ linear transformation T ର matrix ନିର୍ଣ୍ଣୟ କର। ଯେଉଁଠାରେ $\mathbf{B}\mathbb{R}^2$ ଏବଂ $\mathbf{B}\mathbb{R}^3$ ଯଥାକ୍ରମେ $\mathbb{R}^2$ ଏବଂ $\mathbb{R}^3$ ପାଇଁ base ଅଟନ୍ତି । ($\mathbb{R}$ ଏଠାରେ real number ର set କୁ ପ୍ରତିନିଧିତ୍ୱ କରେ।)

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10. The focus of a parabola is $(1,5)$ and its directrix is $x+y+2=0$. Find the equation of the parabola, length of latus rectum and its vertex.

ଗୋଟିଏ parabola ର focus $(1,5)$ ଏବଂ ଏହାର directrix ହେଉଛି $x+y+2=0$ । ତେବେ parabola ର equation, latus rectum ର ଲମ୍ବ ଏବଂ ଏହାର vertex ନିର୍ଣ୍ଣୟ କର।

11. Find the equation to the circle which passes through the points $(2,-2)$, $(3,4)$ and has its center on the line $2x+2y=7$. Find its center and radius.

ଯଦି କୌଣସି ବୃତ୍ତ $(2,-2)$, $(3,4)$ points ମଧ୍ୟଦେଇ ଗତିକରିପାରେ ଏବଂ ଏହାର କେନ୍ଦ୍ର $2x+2y=7$ line ରେ ଅବସ୍ଥିତ, ତାହାର ସମୀକରଣ (equation) ନିର୍ଣ୍ଣୟ କର। ଏହାର କେନ୍ଦ୍ର ଏବଂ ବ୍ୟାସାର୍ଦ୍ଧ ନିଶ୍ଚୟ କର।

12. The vertices of a triangle are $A(4,3,1)$, $B(6,4,3)$ and $C(-8,7,-2)$. The internal bisector of angle A meets $[BC]$ in the point D. Find the coordinate of D and also find the distance of the point A from the point D.

ଗୋଟିଏ ତ୍ରିଭୁଜର ଭୂଲମ୍ବରେଖା (vertices) ହେଉଛି $A(4,3,1)$, $B(6,4,3)$ ଏବଂ $C(-8,7,-2)$। A କୋଣରୁ $[BC]$ କୁ ଆଭ୍ୟନ୍ତରୀଣ ଭାବେ ଦୁଇସମାନ ଭାବରେ ବିଭକ୍ତ କରୁଥିବା ରେଖା D ବିନ୍ଦୁରେ ସ୍ପର୍ଶ କରେ। D ର coordinate ନିର୍ଣ୍ଣୟ କର ଏବଂ D ଠାରୁ A ର ଦୂରତା ସ୍ଥିର କର।

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GROUP—B

ଗ୍ରୁପ୍—B

Candidates to attempt **05 *(five)*** *questions.*
Each question carries **20** *marks.*

ପରୀକ୍ଷାର୍ଥୀମାନେ **୦୫**ଟି ପ୍ରଶ୍ନ **୩୦୦** ଶବ୍ଦ ମଧ୍ୟରେ ଉତ୍ତର ଦେବେ ।
ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନର ମୂଲ୍ୟ **୨୦** ମାର୍କ୍।

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13. Verify Green's theorem for the following integral in xy-plane :

$$\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$$

where c is the boundary of the region bounded by the parabolas $y = \sqrt{x}$ and $y = x^2$.

xy ସମତଳକ୍ଷେତ୍ର (plane) ରେ ନିମ୍ନଲିଖିତ ପାଇଁ ଙ୍କ ପରୀକ୍ଷା କର।

$$\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$$

$y = \sqrt{x}$ ଏବଂ $y = x^2$ ପାରାବୋଲା (parabolas) ଦ୍ୱାରା ସୀମିତ ଅଞ୍ଚଳ ର ସୀମା ହେଉଛି c ।

14. Evaluate

$$\oint_c (xy\, dx + xy^2\, dy)$$

taken round the positivity oriented square with vertices (1,0), (–1,0), (0,1) and (0,–1) using Stokes' theorem and also verify the Stokes' theorem.

ମୂଲ୍ୟାୟନ କର

$$\oint_c (xy\, dx + xy^2\, dy)$$

Stokes' theorem ବ୍ୟବହାର କରି (1,0), (–1,0), (0,1) ଏବଂ (0,–1) ଶୀର୍ଷଗୁଡ଼ିକର (vertices) ସହିତ ସକାରାତ୍ମକତାଭିତ୍ତିକ ବର୍ଗଗୁଡ଼ିକ କିପରେ ଘେ?? ରହିଥାଏ, ଆଲୋଚନା କର। Stokes' ଙ୍କ theorem କୁ ଏଠାରେ ପ୍ରମାଣ କର।

15. Verify Gauss's divergence theorem and show that

$$\iint_S [(x^3 - yz)\hat{i} - 2x^2y\,\hat{j} + 2\hat{k}].\hat{n}\; dS = \frac{a^5}{3}$$

where S denotes the surface of the cube bounded by the plane $x = 0,\ x = a,\ y = 0,\ y = a,\ z = 0,\ z = a.$

Gauss's ଙ୍କ divergence theorem କୁ ପ୍ରମାଣକର ଏବଂ ଦର୍ଶାଅ ଯେ

$$\iint_S [(x^3 - yz)\hat{i} - 2x^2y\,\hat{j} + 2\hat{k}].\hat{n}\; dS = \frac{a^5}{3}$$

ଯେଉଁଠାରେ S ନିମ୍ନୋକ୍ତ ସମତଳ (plane) ଦ୍ୱାରା ବନ୍ଧନଯୁକ୍ତ କ୍ୟୁବ (cube)ର ପୃଷ୍ଠକୁ ସୂଚୀତ କରେ।

$x = 0,\ x = a,\ y = 0,\ y = a,\ z = 0,\ z = a$

16. Find all irreducible polynomials of degree 2 over the field $\mathbb{Z}_3$. Is the polynomial $x^3 + x + 1$, irreducible over the field $\mathbb{Z}_5$? (where $\mathbb{Z}$ represents the set of all integers)

Field $\mathbb{Z}_3$ ରେ ଥିବା degree 2 ର ସମସ୍ତ irreducible polynomials ନିର୍ଣ୍ଣୟ କର। Field $\mathbb{Z}_3$ ରେ polynomial $x^3 + x + 1$କୁ ହ୍ରାସ କରିବା ଅସମ୍ଭବ (irreducible) କି? (ଯେଉଁଠାରେ $\mathbb{Z}$ ସମସ୍ତ ସଂଖ୍ୟାର set କୁ ପ୍ରତିନିଧିତ୍ୱ କରେ।)

17. If V and W be two subspaces of $\mathbb{R}^3$ defined as follows :

$V = \{(a,b,c) : b + 2c = 0\}$ and $W = \{(a,b,c) : a + b + c = 0\}$

Then find bases and dimension of V, W and $V \cap W$.

ଯଦି $\mathbb{R}^3$ର ଦୁଇ subspace V ଏବଂ Wକୁ ନିମ୍ନମତେ ବୁଝିଇ ଦିଆଯାଏ :

$V = \{(a,b,c) : b + 2c = 0\}$ ଏବଂ $W = \{(a,b,c) : a + b + c = 0\}$

ତେବେ V, W ଏବଂ $V \cap W$ ର bases ଏବଂ dimension ନିର୍ଣ୍ଣୟ କର।

18. The equations of the sides AB, BC, CA of a triangle ABC are $3x + 4y = 6$, $12x - 5y = 3$ and $4x - 3y + 12 = 0$. Find the equation of the internal bisector of angle A.

ଗୋଟିଏ ତ୍ରିଭୁଜ ABCର ପାର୍ଶ୍ୱଗୁଡ଼ିକର AB, BC, CA ସମୀକରଣ (equation) ହେଉଛି $3x + 4y = 6$, $12x - 5y = 3$ ଏବଂ $4x - 3y + 12 = 0$। ତେବେ A କୋଣ ର internal bisector ର ସମୀକରଣ ନିର୍ଣ୍ଣୟ କର।

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