CSM – 53/21
Mathematics
Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any ***three*** *of the remaining questions, selecting at least* ***one*** *from each Section.*

SECTION – A

1. Answer any **three** questions of the following :

(a) (i) Find the unique polynomial of degree 2 or less, such that f(0) = 1, f(1) = 3, f(3) = 55 using the Newton divided difference interpolation. 10

(ii) Obtain a linear polynomial approximation to the function $f(x) = x^3$ on the interval [0, 1] using the least squares approximation with w(x) = 1. 10

(b) If a is a simple graph with n (≥ 3) vertices and if deg(v) + deg(w) ≥ n for each pair of non-adjacent vertices v and w, then a is Hamiltonian. 20

(c) (i) Solve $y^2 + (x - \frac{1}{y}) \frac{dy}{dx} = 0$. 10

(ii) Solve $(D^3 - D^2 - 6D)y = 1 + x + x^2$. 10

(d) Find the solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0$ in series about x = 0. 20

2. (a) (i) For the following data, calculate the differences and obtain the forward and backward difference polynomials. Interpolate at x = 0.25 and x = 0.35. 15

x	f(x)
0.1	1.40
0.2	1.56
0.3	1.76
0.4	2.00
0.5	2.28

(ii) Show that the Newton-Raphson method has second order convergence. 15

(b) (i) Derive the first, second and third order differentiation formulae for equidistant nodes based on Newton's forward and backward interpolation formulae. In particular find all the formulae at both the begining and end points tabular values of the independent variable. 15

(ii) Use Runge-Kutta method of order 2 to calculate y(0.2) for the equation : 15

$$\frac{dy}{dx} = x + y^2, y(0) = 1$$

3. (a) (i) Show that if G is a bipartitle graph, then each cycle of G has even length. 15

(ii) Let G be a plane drawing of a connected planar graph and let n, m and f denote respectively the number of vertices, edges and faces of G. Then show that $n - m + f = z$. 15

(b) Let T be a graph with n vertices. Then the following statements are equivalent : 30

(i) T is a tree

(ii) T contains no cycles and has n – 1 edges

(iii) T is connected and has n – 1 edges

(iv) T is connected and each edge is a bridge.

(v) Any two vertices of T are connected by exactly one path.

(vi) T contains no cycles, but the addition of any new edge creates exactly one cycle.

4. (a) (i) Solve $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=e^{-t}$ given $y(0) = 0 = y'(0)$ using Laplace transform method. 15

(ii) Solve : $\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{x^2+y^2}$ 15

(b) (i) Find a complete integral of $q = 3p^2$. 15

(ii) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\cos mx \cos ny$. 15

SECTION – B

5. Attempt any **three** questions of the following :

(a) Draw a flow chart to evaluate

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}\cdots\cdots$$

by direct summation of successive terms, neglecting the first term whose absolute value becomes less than 10^{-6} or adding 20 terms whichever is earlier for a given x in radians.

Also write a program to compute the Euclidean norm of a given real m × n matrix A, where the norm is defined by :

Euclidean norm = $\left(\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2\right)^{\frac{1}{2}}$, take m and $n \leq 10$. 10+10 = 20

(b) Solve the following linear programming problem graphically :

maximize $z = 5x_1 + 7x_2$

Subject to $3x_1 + 8x_2 \leq 12$

$x_1 + x_2 \leq 2$

$2x_1 \leq 3$

$x_1, x_2 \geq 0$ 20

(c) Two equal uniform rods AB, BC each of length 2a and weight w are freely jointed at B and rest in a vertical plane across two smooth horizontal pegs at the same horizontal level and distant a apart. Show that in the position of equilibrium the inclination θ of each rod to the vertical is given by $2 \sin^3\theta = 1$. Determine the magnitude and direction of the reaction at joint B. 10+5+5 = 20

(d) Show that if all the forces in a coplanar system are rotated about their points of application in the same plane thorugh the same angle in the same sense, then their resultant passes through fixed point in the plane. 20

6. (a) Solve the following linear programming problem by simplex method :

Maximize $z = 3x_1 + 5x_2 + 2x_3$

Subject to $x_1 + x_2 + 2x_3 \leq 430$

$3x_1 + x_2 \leq 460$

$x_1 + 4x_3 \leq 420$

$x_1, x_2, x_3 \geq 0$ 30

(b) Test whether the velocity specified by

$$\vec{q} = \frac{k^2\left(x\hat{j} - y\hat{i}\right)}{x^2 + y^2}$$ (k = const), is a possible

motion for an incompressible fluid. If so determine the equation of the stream lines.

Also test whether the motion is of the potential kind and if so determine the velocity potential. 30

7. (a) (i) Determine the image system for a source outside a circle (or a cross section of a circular cylinder) of radius a with the help of the Milne-Thomson Circle Theorem. 15

(ii) Prove that the radius of curvature R at any point of a streamline ψ = constant is given by

$$R = \left(u^2 + v^2\right)^{3/2} \Big/ \left|u^2 \frac{\partial v}{\partial x} - 2uv\frac{\partial u}{\partial x} - v^2 \frac{\partial u}{\partial y}\right|,$$

where u, v are respectively the velocity components of a fluid motion along OX and OY directions, respectively. 15

(b) (i) State and prove D'Alembert's principle and hence deduce the general equations of motion of a rigid body.

5+5+5 = 15

(ii) Determine moment of inertia of a disc of radius a about its diameter OX, say. Also determine the moment of inertia of the disc about a line perpendicular to the disc through the centre O. 7+8 = 15

8. (a) Find the initial basic feasible solution of the following balanced transportation problem using by Vogel's approximations method : 30

	D_1	D_2	D_3	D_4	D_5	a_i	
O_1	4	7	0	3	6	14	
O_2	1	2	−3	3	8	9	**Supply**
O_3	3	−1	4	0	5	17	
b_j	8	3	8	13	8		

Demand

(b) Find out the minimum assignment cost from the following cost matrix : 30

	I	II	III	IV
A	9	6	6	5
B	8	7	5	6
C	8	6	5	7
D	9	9	8	8