CSM – 53/20

Mathematics

Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any **three** of the remaining questions, selecting at least **one** from each Section.*

SECTION – A

1. Attempt any **three** of the following :

   (a) (i) Find the cubic polynomial from the following data by Lagrange's interpolation formual : 10

x	f(x)
0	2

x	f(x)
1	3
2	12
5	147

(ii) Obtain an approximation in the sense of the principle of least squares in the form of a polynomial of degree 2 to the function $1/(1+x^2)$ in the range $-1 \leq x \leq 1$. 10

(b) Prove that a nonempty connected graph G is Eulerian if and only if its vertices are all of even degree. 20

(c) (i) $(1 + y^2)dx = (\tan^{-1} y - x)dy$ 10

(ii) $(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x(1-x^2)^{3/2}$ 10

(d) Find the series solution of differential equation $2x^2y'' - xy' + (1 - x^2)y = 0$. 20

2. (a) (i) The equation $\sin x = 5x - 2$ can be put as $x = \sin^{-1}(5x - 2)$ and also as $x = \frac{1}{5}(\sin x + 2)$ suggesting two iterative procedures for its solution by Iteration method. Which of these, if any, would succeed and which would fall to give root in the neighbourhood of 0.5. 15

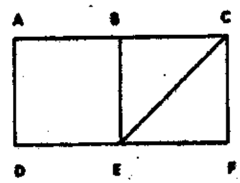
(ii) Derive Newton-Raphson method. Hence prove that the recurrence formula for finding the nth root of a is

$$x_{i+1} = \frac{(n-1)x_i^n + a}{nx_i^{n-1}}.$$ 15

(b) (i) Derive Gauss-Legendre two point formula for numerical integration and obtain the error constant. 15

(ii) Solve the equation $\frac{dy}{dx} = x + y$ with $y(1) = 0$ to obtain $y(1.1)$ and $y(1.2)$ by Taylor series method. 15

3. (a) (i) Consider the Graph G which is given below : 15



Find :

(u) all simple paths from A to F,

(v) all trails from A to F,

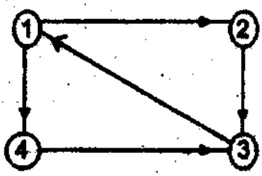
(w) d (A, F); the distance from A to F,

(x) diam(G); the diameter of G,

(y) all cycles which include vertex A,

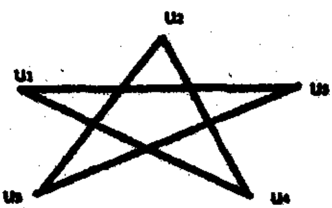
(z) all cycles in G.

(ii) Define the connectedness in a diagraph such as connected or weakly connected, unilaterally connected and strongly connected. Verify whether the directed graph given below is strongly connected ? 15



(b) What do you mean by Isomorphic graphs ? Determine whether the following graphs are isomorphic to each other. If yes, justify your answer. 30

G1



G2

4. (a) (i) Solve $(D^2 + 9)y = \cos 2t$, given that $y = 1$ when $t = 0$, $y = -1$ when $t = \pi/2$, by using Laplace transform method. 15

(ii) $$\frac{dx}{(y^3x-2x^4)}=\frac{dy}{(2y^4-x^3y)}=\frac{dz}{9z(x^3-y^3)}.$$

15

(b) (i) Solve the equation $z^2 = pqxy$ by using Charpit's method. 15

(ii) Solve the partial differential equation $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y} + xy + \sin(2x + y)$, where $D = \frac{\partial z}{\partial x}, D' = \frac{\partial z}{\partial y}$. 15

SECTION – B

5. Attempt any **three** of the following :

(a) Write a flowchart and FORTRAN program to generate Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21,........ 20

(b) Two uniform rods AB, BC of lengths 2a and 2b are rigidly joined at B, and suspended freely from A. If they rest inclined at angles θ, ϕ respectively to the vertical, prove that

$$\frac{a}{b} = \left(1 + \frac{\sin\phi}{\sin\theta}\right)^{\frac{1}{2}} - 1.$$ 20

(c) Find the dual of the LPP : 20

Min $Z = -x_1 + 2x_2 - x_3$,

s. t. $-3x_1 + 2x_2 + 3x_3 = 2$,

$x_1 + 3x_2 + x_3 = 3$, and $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

(d) State and prove parallel axes theorem related with moment of inertia of the body.

20

6. (a) Solve the LP problem : 30

Max $z = 3x_1 + 5x_2 + 4x_3$,

subject to the following constraints

$2x_1 + 3x_2 \leq 8$,

$2x_2 + 5x_3 \leq 10$,

$3x_1 + 2x_2 + 4x_3 \leq 15$

and $x_1, x_2, x_3 \geq 0$

(b) Show that if the velocity field $u(x, y) = \frac{B(x^2 - y^2)}{(x^2 + y^2)^2}$, $v(x, y) = \frac{2Bxy}{(x^2 + y^2)^2}$, $w(x, y) = 0$ satisfies the equations of motion for inviscid incompressible flow, then determine the pressure associated with this velocity field, B being a constant. 30

7. (a) Show that the velocity potential $\phi = \frac{1}{2}\log\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}$ gives a possible motion. Determine the streamlines and also show that the curves of equal speed are the ovals of cassini given by $rr' =$ constant. 30

(b) (i) A body of weight w can just be sustained on a rough inclined plane by a force P and just dragged up the plane by a force Q, P and Q both acting up the line of greatest slope. Show that the co-efficient of friction is $\frac{Q-P}{Q+P}\tan\alpha$, where α is the inclination of the plane to the horizontal. 15

(ii) Show that the necessary and sufficient condition for a system of force to keep a particle in equilibrium, is that the sum of the system components of the forces in three mutually perpendicular directions are independently zero. 15

8. (a) Find the initial basic feasible solution of the following transportation problem using Vogel's approximations methods. 30

Warehouse → Factory ↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

(b) Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and given in the following table : 30

Job

Man

	I	II	III	IV	V
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Find the assignment of the men to jobs that will minimize the total time taken.