CSM – 53 / 16
Mathematics
Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any **three** of the remaining questions, selecting at least **one** from each Section.*

SECTION – A

1. Answer any **four** of the following : $15\times4 = 60$

   (a) Construct Lagrange's interpolation given the following data :

   $$f(0) = 1, f(-1) = 2, f(1) = 3$$

(b) Find the approximate value of the integral $\int_1^2 \frac{dx}{x}$ by the compound Simpson's $\frac{1}{3}$rd rule with 4 sub intervals.

(c) How many edges must a planar graph have if it has 7 regions and 5 vertices. Draw one such graph.

(d) Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw two such graphs.

(e) Find the differential equation of the family of curves $y = Ae^{3x} + Be^{5x}$ for the different values of A and B.

2. (a) Obtain $\sqrt{12}$ to four places of decimal by Newton-Raphson method. 20

(b) Evaluate he integral $I = \int_0^1 \frac{dx}{1+x}$ by subdividing the interval [0, 1] into two equal parts and then applying the Gauss-Legendre three point formula. 20

(c) Applying the Euler method, find the numerical solution of the initial value problem $\frac{dy}{dx} = x + y$; $y(0) = 1$, with spacing $h = 0.2$ on the interval [0, 1]. 20

3. (a) Let G be a simple regular graph with n vertices and 24 edges. Find all possible values of n and give example of G in each case. 20

(b) Prove that a tree T with n vertices has precisely $n - 1$ edges. 20

(c) Let G be a K-critical graph. Prove that G is connected and the degree of every vertex of G is at least $K - 1$. 20

4. (a) Solve : 20

$$y = 2px - xp^2, \text{ where } p = \frac{dy}{dx}.$$

(b) Solve : 20

$$(3 - x)\frac{d^2y}{dx^2} - (9 - 4x)\frac{dy}{dx} + (6 - 3x)y = 0.$$

(c) Find the complete integral of the equation

$$(p^2 + q^2)y = qz, \text{ where } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}.$$

20

SECTION – B

5. Answer any four of the following : 15×4 = 60

(a) At the completion of the following program loops, what will be the values of K, L and M ?

```
      M = 0
      DO  150   I = 1, 10
                K = I
          DO    140  J =  1, 5
                L = J
                M = M + 1
         140    CONTINUE
         150    CONTINUE
```

(b) Find errors, if any, in each computed GO TO Statement :

(i) GO TO (5, 8, 4) MARK

(ii) GO TO (5, 22, 22, 57), KKK

(iii) GO TO (2, 84, 578), ERIK

(iv) GO TO (5, 76, 0, 24), J

(c) If a force P be resolved into two forces making angles 45° and 15° with its direction, show that the latter force is $\frac{\sqrt{6}}{3}P$.

(d) Find the moment of inertia of a uniform circular disk of mass m and radius a about a line through its centre perpendicular to its plane.

(e) Solve the following linear programming problem by graphical method :

Maximize $3x_1 + 2x_2$

Subject to the constraints

$$-2x_1 + x_2 \le 1$$

$$x_1 \le 2$$

$$x_1 + x_2 \le 3$$

and $\quad x_1, x_2 \ge 0$

6. (a) Draw a flow chart and write a Fortran program which caculates $1 + 2 + 3 + \ldots\ldots\ldots\ldots 50$. 20

(b) What is the purpose of a DO Statement ? Write a program to print the first 100 positive integers using DO loop. 20

(c) Write a function subprogram to find the positive, real root (if it exists) of a quadratic equation $ax^2 + bx + c = 0$. The a term is always positive for this problem. If there are no real, positive roots, make the answer equal to 0. 20

7. (a) A light ladder is supported on a rough floor and leans against a smooth wall. How far up the ladder can a man climb without slipping taking place ? 20

(b) A boy standing at the edge of a cliff, throws a ball upward at a 30^o angle with an initial speed of 64 ft/sec. Suppose that when the ball leaves the boy's hand, it is 48 ft above the ground at the base of the cliff. What are the time of flight of the ball and its range ? What are the velocity of the ball and its speed at impact ? ($g = 32$ ft/sec^2). 20

(c) Derive the equation of continuity for incompressible fluid motion. 20

8. (a) Write the dual of the problem : 10

Minimize $Z = 2x_1 + 5x_2$

Subject to $x_1 + x_2 \geq 2$

$2x_1 + x_2 + 6x_3 \leq 6$

$x_1 - x_2 + 3x_3 = 4$

and $x_1, x_2, x_3 \geq 0$

(b) Using simplex algorithm solve the problem : 25

Maximize $Z = 2x_1 + 5x_2 + 7x_3$

Subject to $3x_1 + 2x_2 + 4x_3 \leq 100$

$x_1 + 4x_2 + 2x_3 \leq 100$

$x_1 + x_2 + 3x_3 \leq 100$

and $x_1, x_2, x_3 \geq 0$

(c) Solve the following transportation problem : 25

	S_1	S_2	S_3	S_4	Capacities of sources
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Requirements	20	40	30	10	100