

MATHEMATICS**Paper—II**

Time : Three Hours

Full Marks : 300

*Candidates should attempt Question No. 1 from Section-A and Question No. 5 from Section-B which are compulsory, and any **THREE** of the remaining questions, selecting at least **ONE** from each Section. All questions carry equal marks.*

SECTION—AAttempt any **FIVE** of the following :—

- (a) Obtain the Newton's divided difference interpolating polynomial for the following data and hence find $f(3)$:—

x	:	0	1	2	4	5	6
f(x)	:	1	14	15	5	6	19

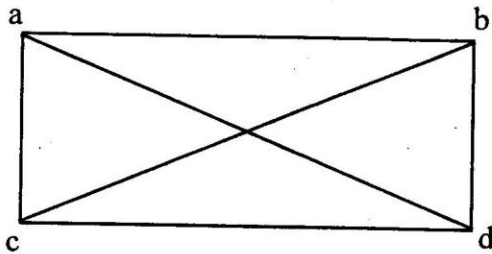
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- (b) If P is the pull required to lift a load W by means of a pulley block, find a linear law $P = mW + c$ from the following data. Here P and W are taken in kg. wt. Compute P when $W = 150$ kg. wt. :—

P	:	12	15	21	25	
W	:	50	70	100	120	12

- (c) Compute $y(0.1)$ and $y(0.2)$ by Taylor's series method if $y' = x^2y - 1$, $y(0) = 1$. 12

- (d) Construct an undirected spanning tree for the connected graph G given below by removing edges in succession. Sketch the graph of the resulting undirected tree.



- (e) Solve by elimination the system of differential equations $x_2' + 2x_1 - x_2 = 1 + e^{-t}$, $x_2' + x_1 + 2x_2 = 3$ subject to the initial conditions

$$x_1(0) = \frac{5}{2} \text{ and } x_2(0) = -\frac{1}{2}. \quad 12$$

- (f) Solve $z(z^2 + xy)(px - qy) = x^4$. 12

2. (a) A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of time t (seconds) :—

t	0.0	0.2	0.4	0.6	0.8	1.0
θ	0.0	0.12	0.49	1.12	2.02	3.20

Calculate the angular velocity and the angular acceleration of the rod when $t = 0.3$ Seconds.

20

- (b) (i) Using Runge-Kutta Second order formula

solve the equation $\frac{dy}{dx} = 2 + \sqrt{xy}$ with

$$y(1) = 1 \text{ for } x = 1 \text{ (0.2) } 1.6. \quad 20$$

- (ii) The following table gives the values of $f(x)$ at equal interval of x :—

x	0	0.5	1.0	1.5	2.0
f(x)	0.399	0.352	0.242	0.129	0.054

Evaluate $\int_0^2 f(x) dx$ using Simpson's rule.

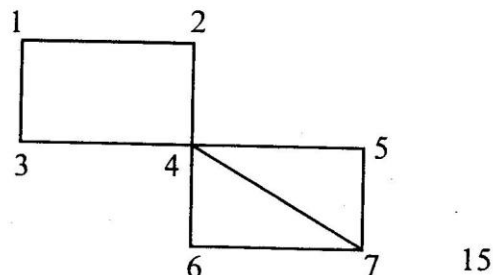
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3. (a) (i) Show that the maximum number of vertices in a binary tree of height n is $2^{n+1} - 1$. 15

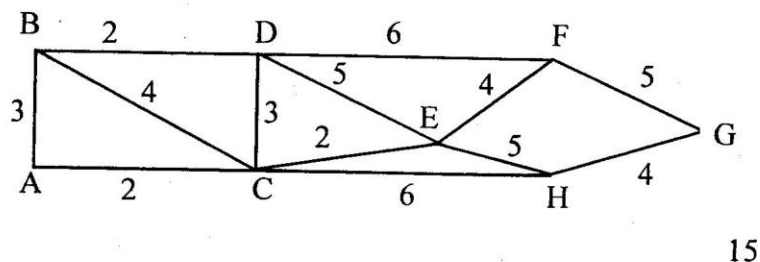
- (ii) Modify Prim's algorithm to handle the case of finding a maximal spanning tree if a certain edge must be included in the tree. 15

- (b) (i) Determine whether the graph shown has a Hamiltonian circuit, a Hamiltonian Path but no Hamiltonian circuit, or neither. If the graph

has a Hamiltonian Circuit, give the circuit :



(ii) Find a Hamiltonian circuit for the graph given :



4. (a) (i) Solve $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^2 + \sin(5 \log x)$. 20

(ii) Solve $\frac{d^2 y}{dt^2} + 4y = u(t - 2)$, where u is the unit step function and $y(0) = 0, y'(0) = 1$. 20

(b) Solve $\frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. 20

SECTION—B

5. Attempt any FIVE of the following :—

(a) Solve the following linear programming problem graphically :

$$\text{Maximize } z = x_1 + x_2$$

subject to the constraints

$$x_1 + x_2 \leq 1$$

$$-3x_1 + x_2 \geq 3$$

$$x_1 \text{ and } x_2 \geq 0.$$

12

(b) A heavy particle is placed on a rough plane inclined at an angle α to the horizon and is connected by a stretched weightless string AP to a fixed point A in the plane. If AB is the line of greatest slope and θ is the angle PAB when the particle is on the point of slipping, show that

$$\sin \theta = \mu \cot \alpha.$$

Interpret the result when $\mu \cot \alpha$ is greater than unity. 12

(c) Prove that the product of inertia of a semi circular wire of mass M and radius a , about diameter and

tangent at its extremity is $2M \frac{a^2}{\pi}$. 12

- (d) A department head has four tasks to be performed and three subordinates who differ in their efficiency. The estimates of the time each subordinate would take to perform each job is given below in the matrix. How should he allocate the tasks one to each man, so as to minimize the total man hours ?

		Men			
		1	2	3	
Task	I	9	26	15	12
	II	13	27	6	
	III	35	20	15	
	IV	18	30	20	

- (e) What is the value of I calculated in the following arithmetic statements ?
- (i) $I = J * 2/3 + K/4 + 6 - J ** 3/8$
($J = 2, K = 5$)
- (ii) $I = B/2. + B * 4./A - B + A ** 3$
($A = 1.5, B = 3.0$)
- (iii) $I = J/2 * 4 + 3/8 + J ** 3$ ($J = 3$).
12
- (f) Draw a flow chart to count non-zero digits in a string of 100 digits.
12

6. (a) A sphere is at rest in an infinite mass of liquid of density ρ , the pressure at infinity being π . Show that, if the radius R of the sphere varies in any manner, the pressure at the surface of the sphere at any time 't' is

$$\pi + \frac{1}{2} \rho \left[\frac{d^2(R^2)}{dt^2} + \left(\frac{dR}{dt} \right)^2 \right]. \quad 30$$

- (b) Two uniform spheres each of mass M and radius a , are firmly fixed to the ends of two uniform thin rods, each of mass m and length l , and the other ends of the rods are freely hinged to a point O . The whole system revolves as in the Governor of a steam engine, about a vertical line through O with angular velocity w . Show that, when the motion is steady, the rods are inclined to the vertical at an angle θ given by the equation

$$\cos \theta = \frac{g}{w^2} \frac{M(l+a) + m \frac{l}{2}}{M(l+a)^2 + m \frac{l^2}{3}}. \quad 30$$

7. (a) What is a subprogram ? Using function subprogram, write a computer program to compute the value of y of a polynomial function
 $y = 4.5 x^3 + 3.1 x^2 - 7.99 x + 1.2$ for values of $x = 1, 2, 3, \dots, 10$.
30

- (b) Solve the following linear programming problem using the principle of duality :

$$\text{Minimize } z = 24x_1 + 30x_2$$

subject to the constraints :

$$2x_1 + 3x_2 \geq 10$$

$$4x_1 + 9x_2 \geq 15$$

$$6x_1 + 6x_2 \geq 20$$

$$x_1, x_2 \geq 0. \quad 30$$

8. (a) A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose centre is O. If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB; show that the velocity of the liquid at any point P is

$$2m \frac{OS^2 + OA^2}{OS} \cdot \frac{PA \cdot PB}{PS \cdot PS' \cdot PT \cdot PT'},$$

where S' and T' are the inverse points of S and T with respect to the circle. 30

- (b) A company manufacturing air-coolers, has two plants located at Mumbai and Kolkata with a weekly capacity of 200 units and 100 units respectively. The company supplies air-coolers to its four showrooms situated at Ranchi, Delhi,

Lucknow and Kanpur which have a demand of 75, 100, 100 and 30 units respectively. The cost of transportation per unit (in rupees) is shown in the following table :—

	Ranchi	Delhi	Lucknow	Kanpur
Mumbai	90	90	100	100
Kolkata	50	70	130	85

Find the transportation schedule so as to minimize the total cost of transportation. 30