राज्य सेवा (मुख्य) परीक्षा - २०२५ परीक्षा दिनांक ०२ मे, २०२६.

2025

T23(B)

BOOKLET NO.
229080

Mathematics

Paper – II

Time Allowed : Three Hours

Maximum Marks : 250

Medium : English

Type of Paper : Conventional / Descriptive

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

1. **There are EIGHT questions divided in two Sections, out of which FIVE are to be attempted.**
2. **Question No. 1 and 5 are compulsory. Out of the remaining, THREE are to be attempted choosing at least ONE question from each Section.**
3. The number of marks carried by a question/sub-question is indicated against it.
4. The medium of answer should be mentioned on the answer book as claimed in the application and printed on admission card. The answers written in medium other than the authorized medium will not be assessed and no marks will be assigned to them.
5. Wherever option has been given, only the required number of responses in the serial order attempted shall be assessed. Unless struck off, attempt of a question shall be counted even if attempted partly. Excess responses shall not be assessed and shall be ignored.
6. Candidates are expected to answer all the sub-questions of a question together. If sub-question of a question is attempted elsewhere (after leaving a few page or after attempting another question) the later sub-question shall be overlooked.
7. Keep in mind the word limit indicated in the question if any.
8. Any page or portion of the page left blank in the Answer Booklet must be clearly struck off.
9. Unless otherwise mentioned, symbol and notation have their usual standard meanings. Assume suitable data, if necessary and indicate the same clearly.
10. Neat sketches may be drawn, wherever required.

Note : Candidates will be allowed to use Scientific (Non-programmable type) calculators.

P.T.O.

SECTION – A

Q1. Solve the following : **(10×5=50)**

Q1. (a) Let G be a group with identity element e and $x \in G$. Prove that the set $S = \{n \in \mathbb{Z} | x^n = e\}$ is a subgroup of the additive group of integers $\mathbb{Z}$. Is S a cyclic subgroup of $\mathbb{Z}$? Justify. **10**

Q1. (b) Let R be a commutative ring with identity whose only ideals are $\{0\}$ and R. Prove that R is a field. **10**

Q1. (c) If $x, y \in \mathbb{R}$ with $x > 0$. Then prove that $\exists\ n \in \mathbb{N}$ such that $nx > y$. **10**

Q1. (d) Prove that a Riemann integrable function on an interval [a, b] in $\mathbb{R}$ is bounded on [a, b]. **10**

Q1. (e) Prove that an analytic function with constant real part is constant. **10**

Q2. (a) Let m, n, a, b be integers and assume that the greatest common of m and n is 1. Prove that there is an integer x such that $x \equiv a$ (modulo m) and $x \equiv b$ (modulo n). **20**

Q2. (b) Prove that a Lipschitz function $f : A(\subseteq\mathbb{R}) \to \mathbb{R}$ is uniformly continuous on A but not conversely. **15**

Q2. (c) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} Z^n$. **15**

Q3. (a) Let $f : [a, b] \to \mathbb{R}$ is monotone on [a, b]. Prove that f is Riemann integrable. **20**

Q3. (b) Prove that every group of order 6 is either isomorphic to the cyclic group of order 6 or to the dihedral group D_3. **15**

Q3. (c) If f(z) is a continuous in a domain D and if for every closed contour C in the domain D $\int_C f(z)dz = 0$, then f is analytic within D. **15**

Q4. (a) For the function $f(z) = \dfrac{2z^2 + 1}{z^2 + z}$, find a Taylor series valid in the neighbourhood of the point $z = i$ and a Laurent series valid within the annulus of which the centre is the origin. **20**

Q4. (b) Let $f(x) = a_n x^n + \ldots + a_0 \in \mathbb{Z}[x]$ be an integer polynomial and let P be a prime integer which does not divide a_n. Prove that if the residue $\overline{f}$ of f modulo P is irreducible, then f is irreducible in $\mathbb{Q}[x]$. **15**

Q4. (c) Prove that the series $\sum_{n=1}^{\infty} \dfrac{1}{n(n+1)}$ converges to 1. **15**

SECTION – B

Q5. Solve the following : **(10×5=50)**

Q5. (a) Prove that the necessary and sufficient condition for a transportation problem to have a solution is that the total demand equals the total supply. **10**

Q5. (b) Find a partial differential equation by eliminating a, b, c from $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{u^2}{c^2} = 1$. **10**

Q5. (c) Find the integral surface of the equation $u_x + y u_y = y\left(\dfrac{e^{-x}}{x}\right)u$ with initial conditions $u = \phi(x)$ for $y = 1$. **10**

Q5. (d) Prove the equality of the Boolean expressions with reasons for $(A + B) \cdot (A + C) = A + B \cdot C$. **10**

Q5. (e) Deduce from D'Alembert's principle the Hamiltonian principle for a conservative system. **10**

Q6. (a) Find the minimum transportation cost for the following transportation costs tableau.

	1	2	3	4	Supply
1	5	9	–	4	28
2	6	10	3	–	32
3	4	2	5	7	60
Demand	48	29	40	33	

The '–' sign in cells (1, 3) and (2, 4) means that the transportation of product units is not possible between the corresponding origins and destinations. **20**

Q6. (b) Find the surface satisfying the equation $(D^2 - 2DD' + D'^2)\,u = 0$ and the condition that $bu = y^2$ when $x = 0$ and $au = x^2$ when $y = 0$. **15**

Q6. (c) Prove that the order of convergence of the Newton-Raphson's iteration method is atleast 2. **15**

Q7. (a) Write down the partial differential equations in two variables in hyperbolic form, parabolic form and elliptic form respectively and distinguish their features. **20**

Q7. (b) Solve the following linear programming problem graphically.

Maximize $Z = 13x_1 + 11x_2$

Subject to
$$4x_1 + 5x_2 \le 1500$$
$$5x_1 + 3x_2 \le 1575$$
$$x_1 + 2x_2 \le 420$$
$$x_1 \ge 0,\ x_2 \ge 0.$$
15

Q7. (c) Solve the following system of equations by Guass-Jordan method.

$$5x - 2y + 3z = 18$$
$$x + 7y - 3z = -22$$
$$2x - y + 6z = 22$$
15

Q8. (a) Using the Euler's method solve $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. Find $y(0.1)$, $y(0.2)$ and $y(0.3)$. **20**

Q8. (b) Verify that the following function satisfies the heat equation :

$u(x, t) = \exp(-\surd^2\alpha^2 t)\cos(\surd x)$, for any real $\surd$. **15**

Q8. (c) Show that the action for the actual path followed by a particle of unit mass in a uniform gravitational field is smaller than any other path. **15**