राज्य सेवा (मुख्य) परीक्षा- २०२५ परीक्षा दिनांक ०२ मे, २०२६.

2025

T22

BOOKLET NO.
156116

Mathematics

Paper – I

Time Allowed : Three Hours **Maximum Marks : 250**

Medium : English **Type of Paper : Conventional / Descriptive**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

1. **There are EIGHT questions divided in two Sections, out of which FIVE are to be attempted.**
2. **Question No. 1 and 5 are compulsory. Out of the remaining, THREE are to be attempted choosing at least ONE question from each Section.**
3. The number of marks carried by a question/sub-question is indicated against it.
4. The medium of answer should be mentioned on the answer book as claimed in the application and printed on admission card. The answers written in medium other than the authorized medium will not be assessed and no marks will be assigned to them.
5. Wherever option has been given, only the required number of responses in the serial order attempted shall be assessed. Unless struck off, attempt of a question shall be counted even if attempted partly. Excess responses shall not be assessed and shall be ignored.
6. Candidates are expected to answer all the sub-questions of a question together. If sub-question of a question is attempted elsewhere (after leaving a few page or after attempting another question) the later sub-question shall be overlooked.
7. Keep in mind the word limit indicated in the question if any.
8. Any page or portion of the page left blank in the Answer Booklet must be clearly struck off.
9. Unless otherwise mentioned, symbol and notation have their usual standard meanings. Assume suitable data, if necessary and indicate the same clearly.
10. Neat sketches may be drawn, wherever required.

Note : Candidates will be allowed to use Scientific (Non-programmable type) calculators.

P.T.O.

SECTION – A

Q1. Solve the following. **(5×10=50)**

Q1. (a) Let $S = \{(1, 3, -4, 2), (2, 2, -4, 0), (1, -3, 2, -4), (-1, 0, 1, 0)\}$ be a subset of $\mathbb{R}^4$ over $\mathbb{R}$. Show that S is linearly dependent subset of $\mathbb{R}^4$. **10**

Q1. (b) Solve the following system of linear equations : **10**

$$3x_1 + 2x_2 + 3x_3 - 2x_4 = 1$$
$$x_1 + x_2 + x_3 = 3$$
$$x_1 + 2x_2 + x_3 - x_4 = 2$$

Q1. (c) Using Mean Value theorem, prove the following :

$1 + x < e^x < 1 + xe^x$, for every $x > 0$. **10**

Q1. (d) Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$. **10**

Q1. (e) Show that the lines $\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3}$, $\frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$ are coplanar and find the equation of the plane containing them.

Also find the equations of the line that intersects the lines $2x + y - 4 = 0 = y + 2z$; $x + 3z = 4$, $2x + 5z = 8$ and passes through the point $(2, -1, 1)$. **10**

Q2. (a) Let T be a linear operator on a finite-dimensional vector space V over field F. Let λ be an eigen-value of T having algebraic multiplicity m. Let E_λ denotes an eigen space of T corresponding to an eigen value λ, then show that $1 \le \dim(E_\lambda) \le m$.

Further, let T be the linear operator on $\mathbb{R}^3$ defined by

$$T\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 4a_1 + a_3 \\ 2a_1 + 3a_2 + 2a_3 \\ a_1 + 4a_3 \end{pmatrix}$$

Then determine the eigen space of T corresponding to each eigen-value. **20**

Q2. (b) Compute $\iint_D \sin[(x-y)/(x+y)]\,dxdy$, where D is the region bounded by $x = 0$, $y = 0$ and $x + y = 1$ in the first quadrant. **15**

Q2. (c) Find the magnitude and equations of the line of shortest distance between the lines :

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7},\ \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$ **15**

Q3. (a) Prove that the improper integral $\int_{\pi}^{\infty} \frac{\sin x}{x}\,dx$ converges conditionally. **20**

Q3. (b) State and prove dimension theorem (rank nullity theorem) for vector spaces. Further, let $T : P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be the linear transformation defined by $T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt$. Comment in detail about one-one ness and onto ness of T. **15**

Q3. (c) i) Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0$, $x - y + z - 2 = 0$, $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere and find its equation. **7**

ii) Show that the sum of the squares of the intercepts made by a given sphere on any three mutually perpendicular straight lines through a fixed point is constant. **8**

Q4. (a) i) Show that a cone can be found so as to contain any two given sets of three mutually perpendicular concurrent lines as generators. **10**

ii) Show that the locus of the line of intersection of tangent planes to the cone $ax^2 + by^2 + cz^2 = 0$ which touch along perpendicular generators is the cone $a^2(b + c)x^2 + b^2(c + a)y^2 + c^2(a + b)z^2 = 0$. **10**

Q4. (b) Let W_1 and W_2 be subspaces of a vector space V.

i) Prove that $W_1 + W_2$ is a subspace of V that contains both W_1 and W_2.

ii) Prove that any subspace of V that contains both W_1 and W_2 must also contain $W_1 + W_2$. **15**

Q4. (c) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. Find the maxima and minima of this function. **15**

SECTION – B

Q5. Solve the following. **(5×10=50)**

Q5. (a) If $L\{f(t)\} = s^{-\frac{1}{2}} \cdot e^{-\frac{1}{s}}$, then find $L\left\{te^{-3t}\, f(4t)\right\}$. **10**

Q5. (b) Solve the differential equation :

$y'' + 3y' + 2y = t$; given that $y(0) = 1,\ y'(0) = -1$. **10**

Q5. (c) A particle is performing a simple Harmonic motion of period T about a centre O and it passes through a point P, where OP = b with velocity v in the direction OP. Then prove that the time which elapses before it returns to P is $\left(\frac{T}{\pi}\right)\tan^{-1}\left(\frac{vT}{2\pi b}\right)$. **10**

Q5. (d) A particle moves in such a way that its acceleration varies inversely as the cube of the distance from a fixed point and is directed towards the fixed point. Discuss the motion. **10**

Q5. (e) Find the flux of $F = yzj + z^2k$ outward through the surface S cut from the cylinder $y^2 + z^2 = a^2$, $z \geq 0$ by the planes $x = 0$ and $x = a$. **10**

Q6. (a) Let $L(y) = y'' + a_1y' + a_2y = 0$ be the linear differential equation with constant coefficients as a_1 and a_2.

Let ϕ_1 and ϕ_2 be two solutions of $L(y) = 0$ on an interval I and $x_0 \in I$. Then show that, ϕ_1, ϕ_2 are linearly independent on I if and only if $W(\phi_1, \phi_2)(x_0) \neq 0$.

Also show that, if ϕ_1, ϕ_2 are any two linearly independent solutions of $L(y) = 0$ on an interval I, then every solution ϕ of $L(y) = 0$ can be written uniquely as $\phi = c_1\phi_1 + c_2\phi_2$, where c_1, c_2 are constants. **20**

Q6. (b) Suppose two bodies are projected from the same point with the same velocity but in different directions. If the range in each case be R and the times of flight be t and t′, then show that $R = \left(\frac{g}{2}\right)tt'$.

Further if h and h′ be the greatest heights in the two paths of a particle with a given velocity for a given range R, then prove that, $R = 4\sqrt{hh'}$. **15**

Q6. (c) Suppose :

$$F = i(e^x \cos y + yz)$$
$$+j(xz - e^x \sin y)$$
$$+k(xy + z).$$

Is F conservative ? If so, find f such that $F = \nabla f$. **15**

Q7. (a) A small bead is projected with any velocity along a smooth circular wire under the action of a force varying inversely as the fifth power of the distance from a centre of force situated on the circumference. Prove that the pressure on the wire is constant. **20**

Q7. (b) Solve the equation :

$$(D^2 - 1)y = x^2 \cdot \cos x.$$ **15**

Q7. (c) Let S be the hemisphere $z = \sqrt{4 - x^2 - y^2}, 0 \le x^2 + y^2 \le 4$, lying above the xy plane, with centre at origin. The boundary of this hemisphere is the circle $C : z = 0,\ x^2 + y^2 = 4$. Show that the integrals $\oint F \cdot dR$ and $\iint_S (\text{curl } F) \cdot n d\sigma$ are equal for S, C and $F = iy - jx$. **15**

Q8. (a) Define what do you mean by a vector field.

Evaluate $\iint z d\sigma$ over the hemisphere $F(x, y, z) = x^2 + y^2 + z^2 = a^2$, $z \geq 0$.

Also integrate $g(x, y, z) = xyz$ over the cube in the first octant bounded by the co-ordinate planes and the planes $x = 1$, $y = 1$, $z = 1$. **20**

Q8. (b) Find $L^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+9}\right\}$ and $L^{-1}\left\{\dfrac{1-e^{-s}}{s^2}\right\}$. **15**

Q8. (c) A particle describes an ellipse under a force to the focus S. When the particle is at one extremity of the minor axis, its kinetic energy is doubled without any change in the direction of motion. Prove that the particle proceeds to describe a parabola. **15**

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