CC(M)
MATHEMATICS
(OPTIONAL)
PAPER - II
[37]

Time Allowed - Three Hours	*Maximum Marks-250*

INSTRUCTIONS

Please read each of the following Instructions carefully before attempting the paper.

i) *There are **Eight** questions divided in **Two** Sections and printed in English. Candidate has to attempt **Five** question in all. Question **No.1** and **5** are compulsory and out of the remaining, any **Three** are to be attempted choosing at least **One** question from each Section. The number of marks carried by a Question/Part is indicated against it. Answers must be written in English in Question-Cum-Answer (QCA) Booklet in the space provided.*

ii) *Your answer should be precise and coherent.*

iii) *If you encounter any typographical error, please read it as it appears in the text book.*

iv) *Candidates are in their Own interest advised to go through the general instructions on the back side of the title page of the Answer Script for strict adherence.*

v) *No continuation sheets shall be provided to any candidate under any circumstances.*

vi) *No blank page should be left in between answers to various questions.*

SECTION - A

1. a) Suppose that H is a subgroup of a group G such that whenever $Ha \neq Hb$ then $aH \neq bH$. Prove that H is a normal subgroup of G. Is the converse true? Justify. **(10)**

b) Show that the series $1+\frac{e^{-2x}}{2^2-1}-\frac{e^{-4x}}{4^2-1}+\cdots$ is uniformly convergent for all real $x \geq 0$. **(10)**

c) If $f(z)$ is an analytic function of $z=(x+iy)$, then prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left|\mathrm{Re}(f(z))\right|^2=2\left|f'(z)\right|^2.$$ **(10)**

d) Prove that a basic feasible solution to a linear programming problem corresponds to an extreme point of the convex set of feasible solutions. **(10)**

e) Prove that the surface whose tangent planes cut off an intercept of constant length a from the z - axis is $z = x\varphi(y/x)+a$, where φ is an arbitrary function. **(10)**

2. a) Let G be a non-Abelian group, and $Z(G)=\{x \in G : xg=gx, \forall g \in G\}$, then prove that

i) $Z(G)$ is a normal subgroup of G, and quotient group $G/Z(G)$ cannot be cyclic.

ii) If H is a normal subgroup of G with $|H| = 2$, then $H \subseteq Z(G)$. **(20)**

b) State Cauchy's integral formula and prove that $\oint_c \frac{\exp(i\pi z)}{2z^2-5z+2}dz=\frac{2\pi}{3}$, where C is the circle $|z|=1$ **(15)**

c) Let $f: [a, b] \rightarrow \mathbb{R}$ be a continuous function. Prove that f is Riemann integrable. **(15)**

3. a) Let R be a ring such that $\{0\}$ and R are the only right ideals of R. Show that either R is a division ring or has prime number of elements such that $ab=0$, $\forall\ a,b \in R$. **(20)**

b) Show that the function $f(x, y, z)=\log(x^2+y^2+z^2)^3-2(x^3+y^3+z^3)$, $(x, y, z)\neq(0, 0, 0)$ has only one extreme value, $\log(3/e^2)$. **(15)**

c) Find three different Laurent series representations for the function $f(z)=\frac{3}{2+z-z^2}$ involving powers of z. **(15)**

4. a) Using contour integration, evaluate $\int_0^{2\pi}\frac{\cos 2\theta}{1-2a\cos\theta+a^2}d\theta$, $-1<a<1$. **(20)**

b) Solve, by simplex method, the system of linear equations

$$x_1+x_2 = 1$$
$$2x_1+x_2 = 4$$ **(15)**

c) Let D be an integral domain. Let $a, b \in D$ be such that $a^m = b^m$, $a^n = b^n$ for two relatively prime positive integers m and n. Prove that $a = b$. Is the result true, if D is not an integral domain? Justify. **(15)**

SECTION - B

5. a) By solving the dual of the following linear programming problem, show that the given problem has no feasible solution.

Minimize $z = x_1 - x_2$

$$\text{Subject to } \begin{aligned} 2x_1 + x_2 &\geq 2, \\ -x_1 - x_2 &\geq 1, \\ x_1, x_2 &\geq 0. \end{aligned}$$ **(10)**

b) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1,\ z = 1$. **(10)**

c) Find the root of the equation $xe^x = 3$ by Regula-Falsi method correct to three decimal places. **(10)**

d) Find the general solution of the partial differential equation

$$(4D^2 - 4DD' + D'^2)\, z = 16 \log(x + 2y), \text{ where } D \equiv \frac{\partial}{\partial x} \text{ and } D' \equiv \frac{\partial}{\partial y}$$ **(10)**

e) Prove that the liquid motion is possible when velocity at (x, y) is given by

$$\left[\frac{ax - by}{x^2 + y^2}, \frac{ay + bx}{x^2 + y^2}, 0\right]$$

Also determine the velocity potential and pressure at any point (x, y). **(10)**

6. a) Find the initial basic feasible solution of the following transportation cost problem by Vogel's approximation method and use it to find its optimal solution. **(20)**

Destination

		A	B	C	D	Availability
Source	S_1	1	2	1	4	30
	S_2	3	3	2	1	50
	S_3	4	2	5	9	20
Demand		20	40	30	10	

b) Solve the following system of linear equations

$$\begin{aligned} 10x_1 + x_2 + x_3 &= 12 \\ 2x_1 + 10x_2 + x_3 &= 13 \\ 2x_1 + 2x_2 + 10x_3 &= 14 \end{aligned}$$

correct up to 4 significant figures by Gauss-Seidel iterative method. **(20)**

c) Write the dual of the following linear programming problem:

maximize $z = 8x_1 + 3x_2 - 2x_3$

$$\text{subject to } \begin{aligned} x_1 - 6x_2 + x_3 &\geq 2 \\ 5x_1 + 7x_2 - 2x_3 &= -4 \end{aligned}$$

$x_1 \leq 0, x_2 \geq 0, x_3$ unrestricted

Also verify that the dual of dual is primal. **(10)**

[Turn Over

7. a) A tank in discharging water through an orifice at a depth of x meter below the surface of the water whose area is A m^2. The following are the values of x for the corresponding values of A.

A	1.257	1.39	1.52	1.65	1.809	1.962	2.123	2.295	2.462	2.650	2.827
x	1.5	1.65	1.80	1.95	2.10	2.25	2.40	2.55	2.70	2.85	3.00

Using Simpson's 1/3 rule in the formula $(0.018)T = \int_{1.5}^{3} \frac{A}{\sqrt{x}}\, dx$, calculate T the time in seconds for the level of the water to drop from 3.0 m to 1.5 m above the orifice. **(20)**

b) i) Write the Boolean expression $(x \wedge (y' \vee z)) \vee z'$ in both disjunctive and conjunctive normal forms.

ii) Let a non-negative integer < 10 be given by its binary representation: $a_3\, a_2\, a_1\, a_0$

If A_i is the statement that a_i is 1, construct a logic circuit corresponding to the condition that the given integer is prime. **(15)**

c) Two sources, each of strength m are placed at the points $(-a, 0)$, $(a, 0)$ and a sink of strength $2m$ at the origin. Show that the stream lines are the curves

$(x^2 + y^2)^2 = a^2 (x^2 - y^2 + \lambda xy)$, where λ is a variable parameter.

Show also that the fluid speed at any point is $\dfrac{2ma^2}{r_1\, r_2\, r_3}$, where r_1, r_2, r_3 are the distances from the sources and the sink. **(15)**

8. a) Find a complete integral and the singular integral of the non-linear partial differential equation given by

$2xy\,(px + qy) + pq = 4\,xyz\,;\; p = \partial z / \partial x,\;\; q = \partial z / \partial y$ **(20)**

b) A hollow cylinder of radius a is fixed with its axis horizontal, inside it rolls without slipping a solid cylinder of radius b, whose velocity in its lowest position is given. If the moving cylinder makes small oscillations about the lowest point of the fixed cylinder, then show that the time period of the small oscillation is $2\pi\sqrt{\dfrac{3(a-b)}{2g}}$.

(15)

c) Using Runge-Kutta method of second order, solve for $y(0.1)$ and $y(0.2)$ given that

$$\frac{dy}{dx} = x^2 - y,\; y(0) = 1$$ **(15)**
