Total No. of Printed Pages-4]  Roll No. ____________

# CC(M)

# MATHEMATICS

# (OPTIONAL)

# PAPER - II

# [37]

*Time Allowed - Three Hours*  *Maximum Marks-250*

## INSTRUCTIONS

***Please read each of the following Instructions carefully before attempting the paper.***

*i)* *There are eight questions divided in Two Sections and printed in English. Candidate has to attempt* ***Five*** *questions in all. Question* ***No.1*** *and* ***5*** *are compulsory and out of the remaining, any* ***Three*** *are to be attempted choosing at least One question from each Section. The number of marks carried by a question/Part is indicated against it. Answers must be written in English in Question-Cum-Answer(QCA) Booklet in the space provided.*

*ii)* *Your answer should be precise and coherent.*

*iii)* *If you encounter any typographical error, please read it as it appears in the text book.*

*iv)* *Candidates are in their own interest advised to go through the general instructions on the back side of the title page of the Answer Script for strict adherence.*

*v)* *No continuation sheets shall be provided to any candidate under any circumstances.*

*vi)* *No blank page should be left in between answers to various questions.*

## SECTION - A

**1.** a) Show that the set $\{4, 8, 12, 16\}$ is a cyclic group under multiplication modulo 20. Find all of its generators. **(10)**

b) Test the convergence or divergence of the infinite series:

$$1^p + \left(\frac{1}{2}\right)^p + \left(\frac{1.3}{2.4}\right)^p + \left(\frac{1.3.5}{2.4.6}\right)^p + \ldots$$ **(10)**

c) Show that the function $f(z) = 2\{\mathrm{Re}(z)\}^2 - i\left(z - \{\mathrm{Im}(z)\}^2\right)$ is nowhere analytic, although, the Cauchy-Riemann equations are satisfied at the origin. **(10)**

d) Find all basic feasible solutions of the set of equations:

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$
$$-x_1 + 2x_2 - 6x_3 + 7x_4 = 3$$

Identify the basic and non-basic variables in each case. **(10)**

e) Verify that the function $u(x, y) = \phi(xy) + x\psi\left(\frac{y}{x}\right)$

is the general solution of the partial differential equation $x^2 u_{xx} - y^2 u_{yy} = 0$, where $\phi$ and $\psi$ are arbitrary twice differentiable functions. **(10)**

**2.** a) Prove that the set $H = \left\{\sigma^2 : \sigma \in S_3\right\}$ is a normal subgroup of the symmetric group $S_3$, the set of all permutations of $\{1, 2, 3\}$. **(20)**

b) Evaluate $\oint_c \frac{z+1}{z^4 + 4z^3}\, dz$, where $C$ is the circle $|z| = 1$. **(15)**

c) Prove that $\int_0^{\pi/2} e^{-R\sin x}\, dx < \frac{\pi}{2R}\left(1 - e^{-R}\right),\ R > 0$ **(15)**

**3.** a) Prove that the series $\frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots$

is non-uniformly convergent on $0 \leq x \leq 1$, but it can still be integrated term by term. **(20)**

b) For any prime $p$, show that the polynomial $f(x) = x^{p-1} - x^{p-2} + x^{p-3} - \ldots + x^2 - x + 1$ is irreducible over the set of rational numbers $\mathbb{Q}$. **(15)**

c) Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for the annular domain $1 < |z-2| < 2$. **(15)**

4. a) Using contour integration, evaluate $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(x^2+9)}\, dx$ **(20)**

b) Solve, by Gauss-Jordan method, the system of linear equations.

$x+2y+z=8$

$2x+3y+4z=20$ **(15)**

$4x+3y+2z=16$

c) Prove that $\frac{F[x]}{\langle x^2+1\rangle}$ and $\frac{F[x]}{\langle x^2+x+4\rangle}$ are isomorphic fields each having 121 elements, where F is the field of integers modulo 11. **(15)**

## SECTION - B

5. a) Find the optimal assignment with minimum cost from the following cost matrix:

**Machines**

| | | I | II | III | IV | V |
|---|---|---|---|---|---|---|
| **Operators** | A | 24 | 29 | 18 | 32 | 19 |
| | B | 17 | 26 | 34 | 22 | --- |
| | C | 27 | 16 | --- | 17 | 25 |
| | D | 22 | 18 | 28 | 30 | 24 |
| | E | 28 | 16 | 31 | 24 | 27 |

Given that the operator B cannot be allowed to operate the fifth machine and similarly the operator C cannot be allowed to operate the third machine. **(10)**

b) Find the solution of the Cauchy problem.

$2xy\frac{\partial z}{\partial x}+(x^2+y^2)\frac{\partial z}{\partial y}=0$ with $z=\exp\left(\frac{x}{x-y}\right)$ on $x+y=1$. **(10)**

c) Using Newton-Raphson method, find the root of the equation $\sin x = 1+x^3$ upto three decimal places. **(10)**

d) Find the general solution of the partial differential equation.

$(D^2-6DD'+9D'^2)z=\tan(y+3x)$, where $D\equiv\frac{\partial}{\partial x}$ and $D'\equiv\frac{\partial}{\partial y}$ **(10)**

e) Prove that the liquid motion is possible when velocity at $(x, y, z)$ is given by

$\left(\frac{3x^2-r^2}{r^5}, \frac{3xy}{r^5}, \frac{3xz}{r^5}\right)$ where $r^2=x^2+y^2+z^2$. Also determine the streamlines.

**(10)**

6. a) Show that the improper integral

$$\int_0^{\pi/2} \sin x \, \log \sin x dx$$

converges and has value $\log 2 - 1$. **(20)**

b) By means of Lagrange's interpolation formula, prove that

$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_{-5})$ (approximately). **(15)**

c) Solve the following linear programming problem

minimize $z = 3x_1 + x_2$

Subject to

$$2x_1 + x_2 \geq 14$$
$$x_1 - x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

by solving its dual problem using simplex method. **(15)**

7. a) The velocity $v$ of a particle at distance $s$ from a point on its path is given in table:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| s(ft) | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
| v(ft/sec) | 47 | 58 | 64 | 65 | 61 | 52 | 38 |

Estimate the time taken in seconds to travel 60ft using Simpson's 1/3 rule. **(20)**

b) A particle of unit mass is constrained to move on the plane $xy = c$ under the action of gravity. Show , by setting up the Lagrange's equation, that $(x^5 + c^2 x)\ddot{x} + 2c^2 \dot{x}^2 - gcx^3 = 0$. **(15)**

c) Using Cauchy's residue theorem, evaluate the integral

$$\int_0^{2\pi} \frac{1}{(2 + \cos\theta)^2} d\theta$$ **(15)**

8. a) Show that the equations

$xp - yq = x, \; x^2 p + q = xz$

are compatible and find their solution. **(20)**

b) Show that the equation of the momental ellipsoid at the corner of a cube of mass M and side $2a$, referred to its principal axes, is given by $2x^2 + 11(y^2 + z^2) =$ constant. **(15)**

c) Using Runge-Kutta method of fourth order, solve for y(0.1) and y(0.2) given that

$\frac{dy}{dx} = x + y^2, \; y(0) = 1$ **(15)**

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