Roll No. ____________

CC(M)
MATHEMATICS
(OPTIONAL)
PAPER - II
[37]

Time Allowed - Three Hours

Maximum Marks-250

INSTRUCTIONS

Please read each of the following Instructions carefully before attempting the paper.

*i) There are eight questions divided in two Sections and printed in English. Candidate has to attempt **Five** questions in all. Questions **No.1** and **5** are compulsory and out of the remaining, any **Three** are to be attempted choosing at least One question from each Section. The number of marks carried by a Question/Part is indicated against it. Answers must be written in English in Question-Cum-Answer (QCA) Booklet in the space provided.*

ii) Your answer should be precise and coherent.

iii) If you encounter any typographical error, please read it as it appears in the text book.

iv) Candidates are in their Own interest advised to go through the general instructions on the back side of the title page of the Answer Script for strict adherence.

v) No continuation sheets shall be provided to any candidate under any circumstances.

vi) No blank page be left in between answer to various questions.

SECTION - A

1. a) Show that the set Q^+ of all positive rational numbers is an abelian group under the binary operation * defined by $a*b=\frac{ab}{3}$, where ab is the ordinary multiplication of two positive rational numbers a and b. **(10)**

b) Test for convergence the series $2x+\frac{3x^2}{8}+\frac{4x^3}{27}+...+\frac{(n+1)x^n}{n^3}+...$ **(10)**

c) Show that the function $f(z)=e^{-z^{-4}}, z\neq 0$ and $f(0)=0$ is not analytic at Z=0 although the Cauchy - Riemann equations are satisfied at that point. **(10)**

d) Test for convergence of the series $1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\frac{30}{33}x^4+...$ **(10)**

e) Form a partial differential equation by eliminating arbitrary functions f and g from $z=f(x^2-y)+g(x^2+y)$ **(10)**

2. a) Let G be a finite abelian group and n a positive integer relatively prime to 0(G). Prove that every $g\in G$ can be written as $g=x^n$ for some $x\in G$. **(20)**

b) Verify Cauchy's theorem for the function 5 sin 2z if C is the square with vertices at $1\pm i, -1\pm i$ **(15)**

c) Examine for term by term integration the series the sum of whose first n terms is $n^2x(1-x)^n (0\le x\le 1)$. **(15)**

3. a) Show that $\frac{1}{2}x^2 < x-\log(1+x) < \frac{1}{2}\frac{x^2}{(1+x)}, if\ -1<x<0$ **(20)**

b) If U is an ideal of R, then prove that $[R:U]=\{x\in R: rx\in U$ for every $r\in R\}$ is an ideal of R and that it contains U. **(15)**

c) Examine the nature of the function $f(z)=\frac{x^2y^5(x+iy)}{x^4+y^{10}}, z\neq 0, f(0)=0$ in a region including the origin. **(15)**

4. a) Solve the following system. **(20)**

$$10x+2y+z=9$$
$$2x+20y-2z=-44$$
$$-2x+3y+10z=22$$

by using Guass - Seidel method.

b) Evaluate $\int_{-\pi}^{2\pi} \frac{a\cos\theta}{a+\cos\theta} d\theta, a>1$ by putting $e^{i\theta}=z$ and using the theory of residues.

(15)

c) Show that $A=\{xf(x)+2g(x): f(x), g(x)\in Z[x]\}$ is not a principal ideal of Z [x] and so Z [x] is not a P.I.D. **(15)**

SECTION - B

5. a) Using simplex algorithm solve the following L.P. problem. **(10)**

$$\text{Minimum } Z=4x_1+8x_2+3x_3$$
$$\text{Subject to} \quad x_1+x_2\geq 2$$
$$2x_1+x_3\geq 5$$
$$x_1, x_2, x_3\geq 0$$

b) Solve by the method of separation of variables.

$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$, If $u(x,o)=x^2(25-x^2)$ **(10)**

c) For the differential equation

$\frac{dy}{dx}=-xy^2$ Find by Runge - Kutta Method of fourth order y(0.6), given that y=1.7231 at x=0.4. Take h=0.2. **(10)**

d) Find the root of log x=cosx, by Newton Raphson method upto five decimal places. **(10)**

e) Show that $\frac{x^2}{a^2}\tan^2 t+\frac{y^2}{b^2}\cot^2 t=1$ is a possible boundary of a liquid motion. Also find the normal velocity. **(10)**

6. a) Show that 'θ'(Which occurs in the Lagrange's mean value theorem) approaches the limit $\frac{1}{2}$ as 'h' approaches zero. Provided that $f''(a)$ is not zero. It is assumed that $f''(x)$ is continuous. **(20)**

b) Apply Lagrange's Formula to find f (5) given that f (1)=2, f(2)=4, f(3)=8, f(4)=16, f(7)=128 and explain why the result differs from 2^5. **(15)**

c) A toy company manufactures two types of doll. a basic version-doll A and a deluxe version - doll B Each doll of type B takes twice as long to produce as one of type A, and the company would have time to make a maximum 2,000 per day if it produces only basic version. The supply of plastic is sufficient to produce 1,500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes profit of Rs. 3.00 and Rs. 5.00 per doll respectively, on doll A and B; how many of each should be produced per day in order to maximize profit? **(15)**

7. a) Three forces P, Q, R acting on a particle are in equilibrium and the angle between P and Q is double of the angle between P and R. Show that. **(20)**

$$P = \frac{Q^2 - R^2}{Q}$$

b) A point moves in a straight line so that its distance from a fixed point in that line is the square root of the quadratic function of the time, prove that its acceleration varies inversely as the cube of the distance from the fixed point. **(15)**

c) Prove by contour integration

$$\int_0^\infty \frac{\log(1+x^2)}{1+x^2}\,dx = \pi \log 2$$ **(15)**

8. a) Find the complete integral of **(20)**

$$Z^2 = pqxy$$

b) A uniform rod OA, of length 2a, free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O, and is inclined at a constant angle α to OZ, show that the value of α is either zero or $\cos^{-1}(3g / 4aw^2)$ **(15)**

c) Given that $\frac{dy}{dx} = \log_e(x+y)$, with the initial condition that y=1 when x=0, find y for x=0.2 and x=0.5, by using Euler's modified method. **(15)**