 Roll No. ____________

CC(M)
MATHEMATICS
(OPTIONAL)
PAPER - I
(37)

Time Allowed - Three Hours | *Maximum Marks-250*

INSTRUCTIONS

Please read each of the following Instructions carefully before attempting the paper.

i) *There are eight questions divided in two Sections and printed in English. Candidate has to attempt* ***Five*** *questions in all. Questions* ***No.1*** *and* ***5*** *are compulsory and out of the remaining, any* ***Three*** *are to be attempted choosing at least One question from each Section. The number of marks carried by a Question/Part is indicated against it. Answers must be written in English in Question-Cum-Answer (QCA) Booklet in the space provided.*

ii) *Your answer should be precise and coherent.*

iii) *If you encounter any typographical error, please read it as it appears in the text book.*

iv) *Candidates are in their Own interest advised to go through the general instructions on the back side of the title page of the Answer Script for strict adherence.*

v) *No continuation sheets shall be provided to any candidate under any circumstances.*

vi) *No blank page be left in between answer to various questions.*

SECTION - A

1. a) Solve $x(1-x^2)dy+(2x^2y-y-ax^3)dx=0$ **(10)**

b) Show that the three vectors (1, 1, –1), (2, –3, 5) and (–2, 1, 4) of R^3 are linearly independent. **(10)**

c) Show that the function f defined on R by.

$$f(x)=\begin{cases} x, \text{when } x \text{ is irrational} \\ -x, \text{ when } x \text{ is rational} \end{cases}$$ is continuous only at x=0 **(10)**

d) Find the work done by the force $F=(2xy+z^3)i+x^2j+3xz^2k$ when it moves a particle from (1, –2, 1) to (3, 1, 4) along any path. **(10)**

e) Find the equation of the sphere that passes through the circle **(10)**

$x^2+y^2+z^2-2x+3y-4z+6=0,$

$3x-4y+5z-15=0$ and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11=0$ orthogonally.

2. a) Find the asymptotes of the curve

$4(x^4+y^4)-17x^2y^2-4x(4y^2-x^2)+2(x^2-2)=0$ and show that they pass through the points of intersection of the curve with the ellipse $x^2+4y^2=4$. **(20)**

b) Solve

$(D^2+a^2)y=\tan ax,$ where D has the usual meaning. **(15)**

c) Show that the vectors (1, 2, 1), (2, 1, 0), (1, -1, 2) form a basis of R^3. **(15)**

3. a) The curve $y^2(a+x)=x^2(3a-x)$ revolves about the axis of x. Find the volume generated by the loop. **(20)**

b) Use the Cayley Hamilton theorem to find the inverse of the matrix. **(15)**

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$$

c) Find the length of the perpendicular drawn from origin to the line $x+2y+3z+4=0=2x+3y+4z+5$

Also, find the equations of this perpendicular and the co-ordinates of the foot of the perpendicular. **(15)**

4. a) Find the rank of $\begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix}$ **(20)**

b) Find the points on the lines

$\frac{x-6}{3}=-(y-7)=z-4$ and $\frac{-x}{3}=\frac{y+9}{2}=\frac{z-2}{4}$ Which are nearest to each other.

Hence find the shortest distance between the lines and also its equations. **(15)**

c) Evaluate $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x^2 dx\, dy\, dz$ **(15)**

SECTION - B

5. a) Find the orthogonal trajectories of the family of Co-axial circles. $x^2+y^2+2gx+c=0$, Where g is the parameter. **(10)**

b) If $u=3x^2y$ and $v=xz^2-2y$, then find

grad $[(grad\, u).(grad\, v)]$ **(10)**

c) Solve $y''-2\tan x . y'+5y=\sec x\, e^x$ **(10)**

d) Find the equation of the plane through the line of intersection of the planes $ax+by+cz+d=0$ and $\alpha x+\beta y+\gamma z+\delta=0$ and Perpendicular to the xy-plane. **(10)**

e) A Point moving in a straight line with uniform acceleration describes distances a,b feet in successive intervals of t_1, t_2 seconds. Prove that the acceleration is $2(t_1 b-t_2 a)/[t_1 t_2(t_1+t_2)]$ **(10)**

6. a) Solve the following system of equations, if consistent-: **(20)**

$$x-4y-z=3$$
$$3x+y-2z=7$$
$$2x-3y+z=10$$

b) Find the equation to the cylinder whose generators are parallel to the line $x=\frac{y}{(-2)}=\frac{z}{3}$, and the guiding curve is the ellipse $x^2+2y^2=1, z=3$. **(15)**

c) Using method of variation of parameters, solve

$$\frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}\right)+y=xe^x\sin x \text{ with } y(0)=0 \text{ and } \left(\frac{dy}{dx}\right)_{x=0}=0.$$ **(15)**

7. a) Find the equations to the line drawn parallel to $\frac{1}{4}x=y=z$ so as to meet the lines $5x-6=4y+3=z$ and $2x-4=3y+5=z$. **(20)**

b) Solve $\left(D^4+D^2+16\right)y=16x^2+256$. where D has the usual meaning. **(15)**

c) A string ABC has its entremities tied to two fixed points A and B is the same horizontal line. To a given point C in the string is knotted a weight W. Prove that the tension in the portion C A is.

$\frac{WB}{4_C\Delta}\left(c^2+a^2-b^2\right)$. Where a,b,c are the sides and Δ is the area of the triangle. **(15)**

8. a) Two heavy particles of weight W and W' are connected by a light inextensible string and hang over a fixed smooth circular cylinder of radius a the axis of which is horizontal. If θ and θ' are the inclinations to the vertical of the radii drawn to the particles, show that $\frac{\text{Sin}\,\theta}{\text{Sin}\,\theta'}=\frac{w'}{w}$ **(20)**

b) Solve

$$6\cos^2 x\left(\frac{dy}{dx}\right)-y\,\text{Sin}\,x+2y^4\,\text{Sin}^3 x=0.$$ **(15)**

c) Test for convergence of the series

$$1+\frac{1}{2}.\frac{x^2}{4}+\frac{1.3.5}{2.4.6.}\frac{x^4}{8}+\frac{1.3.5.7.9}{2.4.6.8.10}\frac{x^6}{12}+......$$ **(15)**