Roll No. ____________

CC(M)

MATHEMATICS

(OPTIONAL)

PAPER-II

[38]

*Time Allowed - **Three** Hours*	*Maximum Marks-**250***

INSTRUCTIONS

Please read each of the following Instructions carefully before attempting the paper.

i) *There are eight questions divided in Two Sections and printed in English. Candidate has to attempt **Five** questions in all. Questions **No.1** and **5** are compulsory and out of the remaining, any **Three** are to be attempted choosing at least One question from each Section. The number of marks carried by a question/Part is indicated against it. Answers must be written in English in Question-Cum-Answer(QCA) Booklet in the space provided.*

ii) *Your answer should be precise and coherent.*

iii) *If you encounter any typographical error, please read it as it appears in the text book.*

iv) *Candidates are in their own interest advised to go through the general instructions on the back side of the title page of the Answer Script for strict adherence.*

v) *No continuation sheets shall be provided to any candidate under any circumstances.*

vi) *No blank page be left in between answer to various questions.*

SECTION - A

1. a) Show that the set of matrices **(10)**

$$A_n = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

where α is a real number, form a group under matrix multiplication.

b) Show that **(10)**

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\ldots+\frac{1}{\sqrt{n^2+n}}\right]=1.$$

c) Suppose $f(z)$ is defined by the integral. **(10)**

$$f(z)=\oint_C \frac{3\zeta^2+7\zeta+1}{\zeta-z}\,d\zeta,$$

where $C : |\zeta| = 3$, then find $f'(1+i)$.

d) Prove the inequality : **(10)**

$$\frac{v-u}{1+v^2}<\tan^{-1}v-\tan^{-1}u<\frac{v-u}{1+u^2}; if\, 0<u<v,$$

and deduce that

$$\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}.$$

e) Find a partial differential equation by eliminating a, b, c from the relation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$ **(10)**

2. a) If H be a normal subgroup of a group G and K a normal subgroup of G containing H, then prove that. **(15)**

$$G/K \cong (G/H)/(K/H).$$

b) If $f(z)=u+iv$ is an analytic function of $z=x+iy$ and $u-v=e^x(\cos y-\sin y)$, find $f(z)$ in terms of z. **(15)**

c) Show that a necessary and sufficient condition for the integrability of a bounded function f is that to every $\varepsilon > 0$, there corresponds $\delta > 0$ such that for every partition P of $[a, b]$ with norm $\mu(P) < \delta$. **(20)**

$$U(P,f)-L(P,f)<\varepsilon$$

3. a) Prove that the volume of the greatest rectangular parallelopiped, that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, is $\frac{8abc}{3\sqrt{3}}$. **(15)**

b) Show that every finite integral domain is a field. **(15)**

c) Prove that the function defined by $f(z)=\begin{cases}\dfrac{x^2y^5(x+iy)}{x^4+y^{10}} & , z\neq 0\\ 0 & , z=0\end{cases}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at the origin. **(20)**

4. a) Using Newton's iterative method, find the real root of equation $x\log_{10}x=1.2$ correct to five decimal places. **(15)**

b) Find the different expansions of $\dfrac{1}{(z-1)(z-3)}$ in powers of z which are valid for regions: **(15)**

i) $|z|<1$ ii) $1<|z|<3$ iii) $|z|>3$

c) Prove that the ring of Gaussian integers is a Euclidean ring. **(20)**

SECTION - B

5. a) Show that there exists no feasible solution of the following linear programming problem: **(10)**

Max $z=3x_1+2x_2$

s.t. $2x_1+x_2\leq 2$

$3x_1+4x_2\geq 12$

and $x,y\geq 0$

b) Find solution of two dimensional Laplace's equation. **(10)**

c) Using Newton's forward difference formula, find the sum $S_n=1^3+2^3+...+n^3$. **(10)**

d) Using Gauss elimination method, solve the equations:

$x+2y+3z-u=10, 2x+3y-3z-u=1, 2x-y+2z+3u=7$ and

$3x+2y-4z+3u=2$. **(10)**

e) For a two dimensional flow the velocities at a point in a fluid may be expressed in the Eulerian coordinates by

$u=x+y+2t, v=2y+t$.

Determine the Lagrange coordinates as functions of the initial position x_0 and y_0, and the time t. **(10)**

6. a) Show that the integral $\int_0^{\infty}\log\sin x\,dx$ is convergent and hence evaluate it. **(15)**

b) The mode of certain frequency curve $y=f(x)$ is very near to $x=9$ and the value of the frequency density $f(x)$ for $x=8.9, 9.0$ and 9.3 are respectively equal to 0.30, 0.35 and 0.25. Calculate the approximate value of the mode. **(15)**

c) Solve the following transportation problem : **(20)**

		A	B	C	D		
	I	21	16	25	13	11	
Source	II	17	18	14	23	13	Availability
	III	32	27	18	41	19	
	Requirement	6	10	12	15	43	

7. a) I. Find the decimal number corresponding to the binary number 1101001.1110011. Also, evaluate the following :

II. Evaluate : $.6756E4 + .7644E6$

III. Evaluate : $.6543E11 \times .5123E-14$ **(15)**

b) A rod, of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describe a cone of semi-vertical angle α; show that $\omega^2 = 3g/(4a \cos \alpha)$. Prove also that direction of reaction at the hinge makes with the vertical an angle

$\tan^{-1}\left(\frac{3}{4}\tan\alpha\right)$. **(15)**

c) Use the method of contour integration to prove that

$$\int_0^{\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2}\, d\theta = \frac{\pi a^2}{1-a^2}\left(a^2<1\right).$$ **(20)**

8. a) Solve : $r+s-6t = y\cos x$. **(15)**

b) The particles of fluid move symmetrically in space with regards to a fixed centre; prove that the equation of continuity is

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{\rho}{r^2}\frac{\partial}{\partial r}\left(r^2 u\right) = 0,$$

where u is the velocity at a distance r. **(15)**

c) Using Runge-Kutta method of fourth order, solve

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with y(0) = 1 at x = 0.2, 0.4. **(20)**
