 Roll No. ____________

CC(M)

MATHEMATICS

(OPTIONAL)

PAPER-I

[37]

Time Allowed - ***Three*** *Hours*	*Maximum Marks*-***250***

INSTRUCTIONS

Please read each of the following Instructions carefully before attempting the paper.

i) There are eight questions divided in Two Sections and printed in English. Candidate has to attempt ***Five*** *questions in all. Questions* ***No.1*** *and* ***5*** *are compulsory and out of the remaining, any* ***Three*** *are to be attempted choosing at least One question from each Section. The number of marks carried by a question/Part is indicated against it. Answers must be written in English in Question-Cum-Answer(QCA) Booklet in the space provided.*

ii) Your answer should be precise and coherent.

iii) If you encounter any typographical error, please read it as it appears in the text book.

iv) Candidates are in their own interest advised to go through the general instructions on the back side of the title page of the Answer Script for strict adherence.

v) No continuation sheets shall be provided to any candidate under any circumstances.

vi) No blank page be left in between answer to various questions

SECTION-A

1. a) Find the orthogonal trajectories of the family of curve $3xy = x^3 - a^3$, where a being parameter of the family. **(10)**

b) Show that the mapping $T: V_3(R) \to V_2(R)$ defined as $T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3, a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(R)$ into $V_2(R)$. **(10)**.

c) Draw the graph of $y = |x-1| + |x-2|$ in the interval $[0,3]$ and discuss the continuity and differentiability of the function in this interval. **(10)**

d) If $\vec{r} = xi + yj + zk$, then prove that $\phi = 1/r$ is a solution of Laplace's equation. **(10)**

e) Two spheres of radii r_1 and r_2 cut orthogonally, then prove that the radius of common circle is $\dfrac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$ **(10)**

2. a) Trace the curve $x^3 + y^3 = 3axy$ **(15)**

b) Find solution of the following Cauchy-Euler equation: **(15)**

$$\left[(3x+2)^2 D^2 + 3(3x+2)D - 36\right] y = 3x^2 + 4x + 1.$$

c) Let T be the linear operator on R^3 defined by **(20)**

$$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$$

What is the matrix of T in the ordered basis $(\alpha_1, \alpha_2, \alpha_3)$ where

$$\alpha_1 = (1,0,1), \alpha_2 = (-1,2,1) \text{ and } \alpha_3 = (2,1,1)?$$

3. a) Curve $y^2(a+x) = x^2(3a-x)$ revolved about the x-axis, find the volume of the solid generated by the loop . **(15)**

b) Find the eigen values of the matrix $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ and verify Cayley- Hamilton theorem for the matrix A. Find inverse of the matrix A and also express $A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A. **(15)**

c) Show that the shortest distance between the lines

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4};\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ and the equation of the line of shortest distance are $\frac{6x-10}{1}=\frac{3y-9}{-1}=\frac{6z-26}{1}$. **(20)**

4. a) For what values of n the equations $x+y+z=1,\ x+2y+4z=n,\ x+4y+10z=n^2$ are consistent and solve them completely in each case. **(15)**

b) Show that the plane $8x-6y-z=5$ touches the paraboloid $3x^2-2y^2=6z$ and find the coordinates of the point of contact. **(15)**

c) Evaluate $\iint(x+y)^2\,dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **(20)**

SECTION-B

5. a) Solve: $(x+y)^2(dy/dx)=a^2$ **(10)**

b) Find constant a,b,c so that $\vec{F}=(x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k$ is irrotational . Also, show that $\vec{F}$ can be expressed as the gradient of a scalar point function. **(10)**

c) Using the concept of inverse Laplace transform, prove that

$$L^{-1}\left\{\tan^{-1}\frac{2}{s^2}\right\}=\frac{2}{x}\sin x.\sinh x.$$ **(10)**

d) Find the point $P\ (x\ ,y,\ z)$ on the plane $2x+y-z=5$ that is closest to the origin. **(10)**

e) A particle moving with simple harmonic motion in a straight line has velocities v_1 and v_2 at distance x_1 and x_2 respectively from the centre of its path. Find the period of its motion. **(10)**

6. a) Prove that every square matrix is uniquely expressible as the sum of a symmetric and a skew symmetric matrix. **(15)**

b) Find the angle between the surfaces $z = x^2 + y^2$ and $z = \left(x - \frac{\sqrt{6}}{6}\right)^2 + \left(y - \frac{\sqrt{6}}{6}\right)^2$ at the point $p = \left(\frac{\sqrt{6}}{12}, \frac{\sqrt{6}}{12}, \frac{1}{12}\right)$. **(15)**

c) Using Laplace transform solve the following initial value problem: **(20)**

$$\left(D^2 + n^2\right)y = a\sin\left(nx + \alpha\right),\ y(0) = y'(0) = 0.$$

7. a) Find the equation of the cone with vertex (α, β, γ) and touching the surface $ax^2 + by^2 + cz^2 = 1$. **(15)**

b) Solve : $\left(D^2 - 1\right)y = \cosh x \cos x$ **(15)**

c) Four rods are joined to form a parallelogram, the opposite points are joined by strings forming diagonals and the whole system is placed on a smooth table. Show that their tensions are in the same ratio as their lengths. **(20)**

8. a) A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If A and B be the base angles and α the angle of projection, then prove that $\tan\alpha = \tan A + \tan B$. **(15)**

b) Apply the method of variation of parameters to solve the equation

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x.$$ **(15)**

c) Verify Green's theorem in the plane for $\int_C \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right]$

Where C is the boundary of the region defined by $y = \sqrt{x};\ y = x^2$. **(20)**
