64th C. C. Exam., 2019

MATHEMATICS

Time Allowed : 3 Hours.] **[Maximum Marks : 300**

*Candidates are required to answer **six** questions in all, selecting **three** from each Section.*

SECTION – I

1. (a) State Cayley-Hamilton theorem and using it find the inverse of the matrix

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

(b) Let $T : V_4 \to V_3$ be a linear map defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3), = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$. Find the rank and nullity of T.

2. (a) Transform the equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ to polar coordinate system.

(b) Evaluate the Euler-Poisson integral $\int_0^\infty e^{-x^2} dx$

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3. (a) Find the centre and radius of the circle

$$x^2+y^2+z^2-2y-4z=11, x+2y+2z=15$$

(b) Find the curvature and torsion of the curve

$$\vec{r}=(a(u-\sin u), a(1-\cos u), bu)$$

4. (a) Solve : $(y+1)\dfrac{dy}{dx}+x(y^2+2y)=x$

(a) Solve : $(D^4-16)y=x^2-e^x, D\equiv\dfrac{d}{dx}$

5. (a) Prove that : $\nabla\times\left(\dfrac{\vec{a}\times\vec{r}}{r^n}\right)=\dfrac{2-n}{r^n}\vec{a}+\dfrac{n(\vec{a}.\vec{r})}{r^{n+2}}\vec{r}$

(b) A covariant tensor has components xy, $2y - z^2$, xz in rectangular coordinates. Find its covariant components in spherical coordinates.

6. (a) Find the intrinsic equation of a common catenary. Find also its equation in cartesian coordinate system.

(b) Define projectile. Find the maximum height and time of flight of a projectile.

SECTION – II

7. (a) Define a group and prove that the set of nth root of unit is a group with respect to multiplication.

(b) If U is an ideal of the ring R, then show that $R|U$ is a ring and is a homomorphic image of R.

8. (a) Define a compact set and show that the continuous image of a compact set is compact.

(b) If $xyz = abc$, then show that the minimum value of $bcx + cay + abz$ is $3abc$

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9. (a) Find the expansion of $f(z)=\frac{7z-2}{(z+1)(z-2)}$ in the region $1<z+1<3$.

(b) Using Charpit's method, solve the following partial differential equation : $z^2 = pqxy$

10. (a) Using regula-falsi method, solve the polynomial equation $x^3 - x + 1 = 0$.

(b) Obtain the cubic spline approximation of the function defined by the data

x :	0	1	2	3
$f(x)$:	1	2	33	144

with $M(0)=0, M(3)=0$.

11. (a) The following data gives the marks obtained by two students A and B in a certain examination :

A :	85	68	64	91	86	82
B :	78	73	76	81	81	83

Find who is more consistent and who is more intelligent.

(b) If X and Y are independent random variables, then show that $E(XY) = E(X)\ E(Y)$

12. (a) Solve the following transportation problem :

Distribution centre

Source	1	2	3	4	
1	2	3	11	7	6
2	1	0	6	1	1
3	5	8	15	9	10
	7	5	3	2	

(b) Solve the dual of the following linear programming problem by simplex method :

Maximize $45x_1 + 80x_2$

Subject to $5x_1 + 20x_2 \le 400$

$10x_1 + 15x_2 \le 450$

$x_1, x_2 \ge 0$

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