63rd C.C. Exam., 2019

MATHEMATICS

Time : 3 Hours] **[Max. Marks : 300**

Candidates are required to answer six questions in all, selecting three from each Section.

SECTION – I

1. (a) If U and W are two sub-spaces of a finite dimensional vector space V, then show that $\dim(U+W) = \dim U + \dim W - \dim(U \cap W)$.

   (b) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

2. (a) If $xyz = abc$, then show that minimum value of $bcx + cay + abz$ is $3\,abc$.

   (b) Evaluate $\iint_E \sin\left(\frac{x-y}{x+y}\right) dxdy$

   where E is the region bounded by the coordinate axes and $x+y=1$ in the first quadrant.

3. (a) If $\vec{r}$ represents the vector joining the points (0, 0, 0) to (x, y, z) and r is its magnitude, then show that the vector $r^n \vec{r}$ is irrotational for every n and solenoidal for $n = -3$.

   (b) Show that the integral $\int_C \vec{F} \cdot dr$, where $\vec{F} = (2x, 2y, 4z)$, is path independent in any domain in space. Find its value, if c has the initial point A : (0, 0, 0) and terminal point B : (2, 2, 2).

4. (a) Solve the differential equation

$$(D^3 - 6D^2 + 11D - 6)y = \cosh x;\ D \equiv \frac{d}{dx}$$

(b) Find a family of curves, which intersects orthogonally to the family $y^2 = cx^3$.

5. (a) Prove that the curve $x = au,\ y = bu^2,\ z = cu^3$ is a helix, if $3ac = \pm 2b^2$.

(b) Find the equation of the sphere containing the circle $x^2 + y^2 + z^2 - x + y + 2z - 4 = 0,\ \ x + 2y - z = 1$ as a great circle.

6. (a) Prove that the least force required to move a body of weight W on a rough horizontal plane is $W \sin\lambda$, where λ is the angle of friction.

(b) A shot after leaving a gun passes just over a wall of a fort horizontally. If the wall is 64 feet high and 192 feet distance from the gun, then find the direction and velocity of projection of the shot.

SECTION – II

7. (a) If ϕ is a homomorphism of a group G onto a group G^* with kernel K, then show that $G|K = G^*$.

(b) Find Laurent series of $f(z) = \dfrac{e^z}{z(1-z)}$ about z = 1. Find the region of convergence.

8. (a) Show that a metric space is compact, if and only if every family of closed sets in it with finite intersection property has nonempty intersection.

(b) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ exists, if and only if m and n are both positive.

9. (a) Solve the partial differential equation $z^2 = pqxy$.

(b) Evaluate the integral $\int_0^1 \frac{dx}{1+x}$ by trapezoidal rule and hence find an approximate value of ln 2.

10. (a) Using Newton-Raphson method, determine a real root of the equation $f(x) = \cos x - xe^x = 0$

such that $|f(x^*)| < 10^{-8}$, where x^* is the approximation to the root.

(b) Using fourth-order classical Runge-Kutta method, solve the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0{\cdot}2$ on the interval $[0, 0{\cdot}4]$.

11. (a) Define binomial distribution and find its mean and variance.

(b) Fit a parabola by applying least square method to the following data :

X :	0·0	0·2	0·4	0·7	0·9	0·10
Y :	1·016	0·768	0·648	0·401	0·272	0·193

12. (a) Using simplex method, solve the following linear programming problem :

Maximize $Z = 3x_1 + 2x_2$

subject to the constraints $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

(b) What is duality in linear programming ? Find the dual of the following linear programming problem :

Maximize $Z = 4x_1 + 2x_2 + x_3$

subject to the constraints

$$x_1 + x_2 + x_3 \geq 3$$

$$x_1 - x_2 \leq 2$$

$$x_1 + 2x_2 + 3x_3 = 4$$

$$x_1 \geq 0$$

$$x_2 < 0$$