of radius r. If 2θ is the angle between the rods, then find the relation between l, r and θ by using the principle of virtual work.

12. (a) Use Stoke's theorem to evaluate the line integral

$$\int_C (-y^3\, dx + x^3\, dy - z^3\, dz)$$

where C is the intersection of the cylinder $x^2 + y^2 = 1$ and plane $x+y+z=1$.

(b) Prove that

$$\text{curl } (\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} - \vec{B} \text{ div } \vec{A} - (\vec{A} \cdot \nabla)\vec{B} + \vec{A} \text{ div } \vec{B}$$

MATHEMATICS, PAPER–II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two from each Section.

SECTION–A

1. (a) Prove that the order of every element of finite group is a divisor of the order of the group.

(b) Prove that every homomorphic image of a group G is isomorphic to some quotient of G.

2. (a) Prove that every integral domain can be embedded into a field.

(b) Prove that in ideal S of the integral domain Z is a maximal ideal of Z iff it is principal ideal generated by a prime number.

3. (a) If (X, d) be a complete metric space and f be a contracting mapping on X, then prove that there exists unique x in X such that $f[x] = x$.

(b) If X is a compact metric space, then prove that a closed subspace of $C(X, R)$ is compact iff it is bounded.

4. (a) Discuss the uniform convergence of the series $\sum_{n=0}^{\infty} x^n (1-x)$ in the interval $\{0, 1\}$.

(b) Divide number n in three parts x, y, z such that the value of $ayz + bzx + cxy$ is maximum or minimum. Determine this value, where a, b, c being the sides of a triangle.

5. (a) Prove that a power series represents an analytic function inside its circle of convergence.

(b) Prove that

$$\int_0^{2\pi} \frac{\sin^2 \theta}{a + b\cos\theta} \, d\theta = \frac{2\pi}{b^2} \left\{a - \sqrt{(a^2 - b^2)}\right\}, \text{ if } a > b > 0$$

6. (a) If $f(z)$ be analytic within and on a simple closed contour C, then prove that $|f(z)|$ attains its maximum value on C unless $f(z)$ is constant.

(b) Prove that $\int_0^{\infty} \frac{x^6}{(a^4 + x^4)^2} dx = \frac{3\pi\sqrt{2}}{16a}, \; a > 0$

7. (a) Find the partial differential equation by eliminating the arbitrary function ϕ from the equation

$$\phi(x + y + z, \; x^2 + y^2 - z^2) = 0$$

Also, solve the partial differential equation

$$(z^2 - 2yz - y^2) \, p + x(y + z)q = x(y - z)$$

(b) Solve : $z - xp - yq = a\sqrt{(x^2 + y^2 + z^2)}$

8. (a) Solve :

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\,\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=(x^2+y^2)^{n/2}$$

(b) Solve the following differential equation by Charpit's method :

$$2(pq+py+qx)+x^2+y^2=0$$

SECTION–B

9. (a) A solid body of density ρ is in the shape of the solid formed by the revolution of the cardioid $r=a(1+\cos\theta)$ about the initial line. Prove that its moment of inertia about a straight line through the pole perpendicular to initial line is

$$\frac{352}{105}\pi\rho a^2$$

(b) A rod of length $2a$ is suspended by a string of length l attached to one end. If the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, then prove that

$$\frac{3l}{a}=\frac{(4\tan\theta-3\tan\phi)\sin\phi}{(\tan\phi-\tan\theta)\sin\theta}$$

10. (a) A homogeneous solid hemisphere of mass M and radius a rests with its vertex in contact with a rough horizontal plane. A particle of mass m is placed on its base which is smooth at a distance c from the centre C. Prove that the hemisphere will commence to roll or slide according as the coefficient of friction.

$$\mu > \text{or} < \frac{25\,mac}{26(M+m)a^2+40\,mc^2}$$

(b) Three equal uniform rods AB, BC and CD hinged freely at B and C are being on a smooth horizontal table so that ABC and BCD are perpendicular on opposite sides of BC. A blow is given to A in the direction AC. Prove that D begins to move in a direction $\tan^{-1}\left(\frac{7}{4}\right)$ with DC.

11. (a) Find the roots of the quadratic equation $x^2 - 5x + 2 = 0$ correct to four decimal places by the Newton-Rephson method.

(b) Find the values of A_1, A_2 and x_1, x_2 such that quadrature formula $\int_{-1}^{1} f(x)\, dx = A_1\, f(x_1) + A_2 f(x_2)$ may give correct result for all polynomials of degree less than or equal to 3.

12. (a) Using Runge-Kutta method, solve the equation $\frac{dy}{dx} = x + y$ with initial condition $y(0) = 1$ from $x = 0{\cdot}1$ to $x = 0{\cdot}4$, when $h = 0{\cdot}1$.

(b) A and B throw alternatively a pair of dice. A wins, if he throws 6 before B throws 7 and B wins, if he throws 7 before A throws 6. If A begins, prove that the chance of A's winnings is $\frac{30}{61}$.

13. (a) Explain the concepts of correlation and regression. How do they differ from each other ? Why are there two lines of regression ?

(b) State and compare the distinctive features of the Binomial, Poisson and Normal probability distributions. When does a Binomial distribution tend to become a Normal and a Poisson distribution ?

14. (a) In a distribution, the first four moments about the value 4 are $-1{\cdot}5$, 17, -30 and 108 respectively. Calculate the four

moments about the mean and about the origin. Find the coefficient of variation and determine the values of β_1 and β_2, and comment upon skewness and kurtosis.

(b) What is chi-square test ? What are the conditions for its application ? Name three situations where this test is used.

15. (a) Solve the following unbalanced transportation problem :

	X	Y	Z	a_1
A	7	3	6	5
B	4	6	8	10
C	5	8	4	7
D	8	4	3	3
b_j	5	8	10	

(b) If the linear programming problem Maximize $Z = CX$ such that $AX = b,\ X \geq 0$ admits an optimal solution, then prove that at least one basic feasible solution must be optimal.

16. (a) Prove that every extreme point of the convex set of all feasible solutions of the system $AX = b$ is a basic feasible solution and vice versa.

(b) Solve the dual of the following problem :

Minimize $\quad Z = 100\,x_1 + 100\,x_2 + 100\,x_3$

subject to

$$2x_1 + 2x_2 + x_3 \geq 22$$
$$2x_1 + x_2 + 2x_3 \geq 30$$
$$x_2 + 2x_3 \geq 25$$

and $\quad x_1, x_2, x_3 \geq 0$

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