56-59th C.C. Exam., 2016

MATHEMATICS, PAPER–I

Time : 3 Hours] [Max. Marks : 200

Answer any five questions.

1. (a) Prove that a subset of linearly independent set of a vector space is linearly independent.

(b) Using row reduction method, find the inverse of the following matrix :

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & 0 & 1 & 1 \\ 1 & 2 & 3 & 0 \\ 3 & -1 & 0 & 1 \end{bmatrix}$$

2. (a) Find the eigenvalues and the eigenvectors corresponding to each eigenvalue of the following matrix :

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 2 \end{bmatrix}$$

(b) Let V and W be the following subspace of R^4 :

$$V = [(a_1, a_2, a_3, a_4) : a_2 - 2a_3 + a_4 = 0]$$

and $W = [(a_1, a_2, a_3, a_4) : a_1 = a_4,\ a_2 = 2a_3]$

Find a basis and the dimension of (i) V, (ii) W, (iii) $V \cap W$.

3. (a) Find the asymptotes of the following curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1$$

(b) Evaluate :

$$\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\, dx\, dy\, dz$$

4. (a) For the function $f(x) = \begin{cases} \dfrac{e^{1/x}}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ test the continuity at $x = 0$.

(b) Using the Lagrange's mean value theorem, prove that for the parabola $y = ax^2 + bx + c$, the abscissa of the point at which the tangent is parallel to given chord is same as the abscissa of the middle point of the chord.

5. (a) If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$.

(b) Find the minimum value of $x^2 + y^2 + z^2$ when $ax + by + cz = p$.

6. (a) Evaluate the following :

(i) $\int_0^{\infty} x^{m-1} e^{-ax} \cos bx\, dx$

(ii) $\int_0^{\infty} x^{m-1} e^{-ax} \sin bx\, dx$

(b) Trace the following cardioid :

$r = a\,(1 + \cos\theta)$

7. (a) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is

$$\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$$

(b) Find the pole of the plane $lx + my + nz = p$ with respect to the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

8. (a) Solve :

$$x\,dx + y\,dy = a^2\left(\frac{x\,dy - y\,dx}{x^2+y^2}\right)$$

(b) Solve :

$$(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

9. (a) Solve : $(x-y)\,dy = (x+y+1)\,dx$

(b) Solve : $(D^2 + 4D - 12)y = (x-1)e^{2x}$

10. (a) Explain Kepler's law of planetary motion. What will be the time period of that planet whose radius of orbit around the sun is double that of earth ?

(b) The angular elevation of an enemy's position on a hill h feet high is β. Show that in order to shell it, the initial velocity of the projectile must not be less than

$$\sqrt{\{gh(1+\operatorname{cosec}\beta)\}}$$

11. (a) Show that if the displacement of a particle moving in a straight line is expressed by the equation $x = a \cos nt + b \sin nt$, it describes a simple harmonic motion whose amplitude is $\sqrt{a^2+b^2}$ and period is $\frac{2\pi}{n}$.

(b) Two equal uniform rods AB and AC, each of length l, are freely jointed at A and rest on a smooth fixed vertical circule