

MATHEMATICS, PAPER-II

Time Allowed : 3 Hours]

[Max. Marks : 200

Answer any five questions, selecting at least two question from each Section.

SECTION-A

1. (a) Show that the set S_A of all permutations of a non-void set A is a group for the product of permutations.
(b) If a is an element of a group G and its normalizer $N(a) = \{x \in G / ax = xa\}$ then prove that $N(a)$ is a subgroup of G .
2. (a) State and prove Sylow's first theorem.
(b) If R is a unique factorization domain (UFD) then the polynomial ring $R[x]$ is also a unique factorization domain.
3. (a) Prove that a metric space X is compact if it is both complete and totally bounded.
(b) Show that the integral function ϕ of a continuous function f in $[a, b]$ is differentiable and $\phi'(x) = f(x) \cdot \forall x \in (a, b)$.
4. State and prove Implicit function theorem.
5. (a) Let G and Ω be open subsets of $\mathbb{C}$. Suppose that $f: G \rightarrow \mathbb{C}$ and $g: \Omega \rightarrow \mathbb{C}$ are continuous functions such that $f(G) \subset \Omega$ and $g(f(z)) = z$ for all z in G . If g is differentiable and $g'(z) \neq 0$, f is differentiable and $f'(z) = \frac{1}{g'(f(z))}$. If g is analytic, f is also analytic.
(b) Let f be analytic in the disk $B(a, R)$ and suppose that γ is a closed rectifiable curve in $B(a, R)$. Then prove that

$$\int_{\gamma} f(z) dz = 0.$$

6. Apply Cauchy's Residue theorem, prove that :

$$(a) \int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}} \quad (b) \int_0^{\pi} \frac{d\theta}{a + \cos \theta} = \frac{\pi}{\sqrt{a^2 - 1}}.$$

7. (a) Form a partial differential equation by eliminating the function f from the following : $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

- (b) Solve : $z - px - qy = a\sqrt{x^2 + y^2 + z^2}$.
8. (a) Apply Charpit's method to find complete integral of

$$2xz - px^2 - 2qxy + pq = 0.$$
- (b) Solve : $(D^2 - 2DD^1 - 15D^{12})z = 12xy.$

SECTION-B

9. (a) Find the moment of inertia of a hollow sphere about a diameter, its external and internal radii being a and b .
- (b) find the generalized forces for the generalized co-ordinates describing the small amplitude oscillation of a double pendulum. Use D'Alembert's principle to find the equation of motion.
10. (a) Find the equation of Vertex-motion for an incompressible fluid.
- (b) State and prove Bernoulli's theorem.
11. (a) Find the root of the equation $x^3 - 9x + 1 = 0$ between $x = 2$ and $x = 4$ by the method of bisection.
- (b) Find the real root of the equation $x \log_{10} x - 1.2 = 0$ correct to five place of decimal by using Regula-Falsi method.
12. (a) Find $f'(0.4)$ from the following table :

x	.01	.02	.03	.04	.05	.06
$f(x)$	.1023	.1047	.1071	.1096	.1122	.1148

- (b) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 1]$. Use the fourth order classical Runge-Kutta method.
13. (a) Calculate the first four moment of the following about the mean and hence find β_1 and β_2 :

x	0	1	2	3	4	5	6	7
f	1	8	28	56	70	56	28	8

- (b) If two dice are thrown, what is the probability that the sum is
 (i) greater than 8, and (ii) neither 7 nor 11 ?
14. (a) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as

Poisson variate with mean 1.5. Calculate the proportion of days on which

(i) neither car is used and (ii) some demand is refused.

(b) Fit a parabola of second degree to the following data :

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

15. (a) Use duality to solve the following L.P.P. :

Maximize $z = 2x_1 + x_2$
 subject to $x_1 + 2x_2 \leq 10$
 constraints $x_1 + x_2 \leq 60$
 $x_1 - x_2 \leq 2$
 $x_1 - 2x_2 \leq 1$
 and $x_1 \geq 0, x_2 \geq 0$.

(b) Use dominance property to solve the following game whose

pay-off matrix is :

	3	2	4	0
	3	4	2	4
	4	2	4	0
	0	4	0	8

16. (a) The time spent by a T.V. repairman on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson at an average rate of 10 per 8 hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

(b) A firm is considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value Rs. 200. The running (maintenance & operating) costs in rupees are found from experience to be as follows :

Year	1	2	3	4	5	6	7	8
Running cost	200	500	800	1200	1800	2500	3200	4000