48-52 C.C. Exam., 2009

MATHEMATICS, PAPER–I

Time Allowed : 3 Hours] **[Max. Marks : 200**

Answer any five questions.

1. (a) Find the characteristics equation of the matrix : $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. Is Cayley-Hamilton theorem verified for this matrix ?

   (b) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

2. (a) Find the rank of the matrix : $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ by reducing it to normal form.

   (b) Reduce the following in quadrative from and find its rank and signautre :

$$x^2 + 4y^2 + 9z^2 + t^2 - 12yz + 6zx - 4xy - 2xt - 6zt.$$

3. (a) A write of length 20 metres is bent so as to form a circular sector of maximum area. Find the radius of the circular sector.

   (b) If $f(x, y) = \log(3x_2 + 4y_2 + 2x + 7)$,

   (i) Determine the region D in which f is defined.

   (ii) Find and classify all critical points in D.

   (iii) Find the maximum and minimum of f in the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$.

4. (a) Evaluate $\int x^2 \log(1-x^2)\,dx$ and deduce that

$$\frac{1}{1\cdot 5}+\frac{1}{2\cdot 7}+\frac{1}{3\cdot 9}+\ldots\ldots=\frac{8}{9}-\frac{2}{3}\log 2.$$

(b) Find the volume of the solid generated by the revolution of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ about y-axis.

5. (a) Show that the two straight lines $x^2(\tan^2\theta+\cos^2\theta)-2xy\tan\theta+y^2\sin^2\theta=0$ make with the x-axis angles such that the difference of their tangents is 2.

(b) Find the equation and length of the common tangent to the hyperbolas : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1$.

6. (a) Prove that the locus of the lines which intersect the three lines $y-z=1, x=0; z=1, y=0; x-y=1, z=0$ is $x^2+y^2+z^2-2yz-2zx-2xy-1=0$. Prove also that of all these lines are just two which intersect the line $y=z-x$ as well and that these two are inclined at an angles $\cos^{-1}\frac{3}{8}$.

(b) Find the equations to the generating lines of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, passing through the point 'θ, ϕ'.

7. (a) Solve : $(10y-x-21)\,dy-(5x-14y-3)\,dx=0$.

(b) Solve : $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^2$ if $y=\frac{3}{8}, \frac{dy}{dx}=1$ when $x=0$.

8. (a) Solve : $(x+a)^2\frac{d^2y}{dx^2}-4(x+a)\frac{dy}{dx}+6y=x$.

(b) Solve the following differential equation :

$$(D^2-2D+1)\,y=x^2+e^{3x}.$$

9. (a) If the position vectors of A, B, C are $2\hat{i}+4\hat{j}-\hat{k}, 4\hat{i}+5\hat{j}+\hat{k}$ and $3\hat{i}+6\hat{j}-3\hat{k}$ respectively, show that triangle ABC is a right angled triangle.

(b) Verify Stokes' theorem for vector function $y\hat{i} + z\hat{j} + x\hat{k}$ and the surface S given by $x^2 + y^2 + z^2 = 1$ and $z \geq 0$.

10. (a) Find the covariant and contravariant components of the acceleration vector in cylindrical and spherical co-ordinates.

(b) If p_1, ρ_1, t_1; ρ_2, t_2; p_2, ρ_2, t_2; p_3 ρ_3 t_3 be the corresponding values of the pressure density and temperature of the same gas, show that

$$t_1\left(\frac{p_2}{\rho_2}-\frac{p_3}{\rho_3}\right)+t_2\left(\frac{p_3}{\rho_3}-\frac{p_1}{\rho_1}\right)+t_3\left(\frac{p_1}{\rho_1}-\frac{p_2}{\rho_2}\right)=0.$$

11. (a) A heavy uniform rod of length $2a$ rests with its ends in contact with two smooth inclined planes of inclinations α and β to the horizon. If θ be the inclination of the rod to the horizon, prove by the principal of virtual work that $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$.

(b) Two forces act, onealong the line $y = 0$, $z = 0$ and the other along the line $x = 0$, $z = c$. As the forces vary, show that the surface generated by the central axis of their equivalent wrench is $(x^2 + y^2)\, z = cy^2$.

12. (a) A heavy particle is projected with velocity u strikes at angle of 45° to an inclined plane of angle β which passes through the point of projection. Show that the vertical height of the point struck above the point of projection is

$$\frac{u^2}{g}\left(\frac{1+\cot\beta}{2+2\cot\beta+\cot^2\beta}\right).$$

(b) A heavy particle is projected with velocity v at an inclination α to the horizon in a medium in which there is resistance equal to k times the velocity of the particle. Discuss the motion and prove that the direction of motion will again make an angle α with the horizon after a time $\frac{1}{k}\log\left(1+\frac{2kv}{g}\sin\alpha\right)$.