$\frac{1}{5}$ units. Obtain the equation of the orbit in the form : $r = 3 + 2\cos\theta$.

12. (a) A uniform elliptic lamina, whose axes are $2a$ and $2b$, is half immersed in water, the axis $2b$ being in the surface. Find the centre of pressure.

(b) A uniform rod of length $2a$ can turn freely about one end, which is fixed at a height h ($< 2a$) above the surface of a liquid; if the densities of the rod and liquid be l and σ, show that the rod can rest either in a vertical position or inclined at an angle θ to the vertical such that $\cos\theta = \frac{h}{2a}\sqrt{\frac{\sigma}{\sigma - \rho}}$.

(c) Prove that : $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r^2 = x^2 + y^2 + z^2$.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Show that a cyclic group of order n has one and only one subgroup of order d for every positive divisor d on n. let $S = \{1, w, w^2, -1, -w, -w^2\}$ where $w = \cos\frac{2\pi}{2} + i\sin\frac{2\pi}{3}$ prove that S is a cyclic subgroup under multiplecation.

(b) If p be a prime and a be an interger such that p is not a divisor of a, then prove that $a^{p-1} = 1 \pmod p$.

2. (a) Let $\phi : \{G, o\} \rightarrow \{G' *\}$ be an isomorphism, then prove that
(i) G', is commutative if and only if G is commutative
(ii) G' is cyclic if and only if G is cyclic.

(b) Prove that a fenite division ring is a field. Also prove that if p be a prime number then p is a divisor of $(p-1)! + 1$.

3. (a) Let $| \leq p \leq \infty$. Prove that $\| a + b \|_p \leq \| a \|_p + \| b |_p$ for $a, b \; E \; K^n$. Further prove that equality occures if there exist constants C_1, C_2 such that $C_1 a = C_2 b$.

(b) Prove that any continuous function from a compact metric space to any other matrix space is uniformly continuous.

4. (a) A function f is defined on $[a, b]$ by $(x) = e^x$.

Find $\int_a^{-b} f$ and $\int_{-a}^{-b} f$. Deduce that F is intecpable on $[a, b]$.

(b) A function $f[a, b] \to R$ is integrable on $[a, b]$ prove that $|f|$ is integrable on $[a, b]$.

5. (a) State and prove cancly' sintegral formula.

(b) Apply caciculas of residue, prove that

$$I = \int_0^{2\pi} \frac{\sin^2 \theta\, d\theta}{a + b\cos\theta} = \frac{2\pi}{b^2}\left[a\sqrt{(a^2 - b^2)}\right], \text{ where } a > b > 0.$$

6. (a) If a function $f(2)$ is continuous in a simply connected region R and $\int_C f(2) dz = 0$ for every closed contoure within R, than prove that $F(2)$ is analytic through out R.

(b) Prove that $I = \int_0^{\infty} \frac{x \sin ax\, dx}{x^2 + b^2} = 1/2\, \pi e^{-ab}$, where $a > \| o$ and b is real.

7. (a) Form a partial differental equation by eliminating the arbitrary function ϕ from $\phi\,(x + y + z, x^2 + y^2 - z^2) = 0$. What is the order of this partial differential equation ?

(b) Solve $(x^2 - y^2 - z^2)\, p + 2xyq = 2xz$.

8. (a) Find a complete, singular and general integrals of $(p^2 + q^2) y = qz$.

(b) Solve $z = px + qy + c\sqrt{(1 + p^2 + q^2)}$.

SECTION–B

9. (a) Derive lagrange's equations for constrained motion. Can you derive Euler's dynamical equations from lagrange's equations under finite forces ?

(b) Show that the total kinetic energy of a rigid body of mass M moving in two dimensions is equal to the kinetic energy of the particle of mass M placed at centre of inertia and moving with it plus the kinetic energy of the body relative to the centre of inertia.

10. (a) लैग्रांजियन तथा आइलेरियन प्रारूप के सतता के समीकरणों की समतुल्यता दिखाइये।

(b) Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$ is a possible form of the boundary surface of a liquid and find an expression for the normal velocity.

11. (a) Derive Bernouli's equation for unsteady irrofational motion of an incompressible fluid.

(b) What arrangement of sources and sinks will give rise to the function $w = \log z\left(z - \frac{a^2}{z}\right)$. Find an expression for stream lines.

12. (a) Explain Newton-Raphson method. Use this method of find a roof of tac equation $x^3 - 2x - 5 = 0$ an discuss its convergence.

(b) Define a cubic spinen, tridiagonal system and find a natural cubic spene to the following data and compute

(i) y (1.5 and (ii) $y = 1$.

x	1	2	3
y	-8	-1	18

13. (a) Show that for any discrete distribution, the standard deviation is not less than the mean deviation from the mean.

(b) Explain moments for Bivariate distribution. Prove that if the *Kth* moment of X exists, then all the moments of order less than *K* exist.

14. (a) A box contain 2^n tickets among which nc_i tickets bear the number i ($i = 0, 1, 2, \ldots, n$). A group of m tickets is brawn. Let S denote the sum of their numbers. Find $E(s)$ and Var S.

(b) Explain pearson's distribution. Let X and Y be independent variates with the common on cauchy density.

$$\frac{1}{\pi}\cdot\frac{1}{1+x^2}, -\infty < x < +\infty.$$

Prove that $u = XY$ has density $f(u) = \frac{2}{\pi^2}\left\{\frac{\log(u)}{u^2 - 1}\right\} - \infty < u < \infty$.

15. (a) What are transportation problems ? Solve the transportation problem with cost coefficients, demands and supplies as given in the following table,

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	−2	3	70
O_2	2	4	0	1	38
O_3	1	2	−2	5	32
Demand	40	28	30	42	

(b) Explain Bellman's optimality principle. Solve the following problem by Dynamic programming.

$$\text{Maximize } \sum_{n=1}^{4}(4U_n - nU_n^2)$$

$$\text{Subject to the constraints : } \sum_{n=1}^{4} u_n = 10,\ u_n \geq 0.$$

16. (a) At what average rate mist a clerk at a super market work in order to insure a probability of 0.90 that the customer will not have to wait longer than 12 minuts ? It is assumed that there is only one counter, to which customers arrive in a poisson fashion at an average rate of 15 per hour. The length of service by the clerk has an exponential distribution.

(b) Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information (processing time on machinesis given in hours and passing in not allowed).

Job	A	B	C	D	E	F	G
Machinine M_1 :	3	8	7	4	9	8	7
Machinine M_2 :	4	3	2	5	1	4	3
Machinine M_3 :	6	7	5	11	5	6	12

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MATHEMATICS, PAPER – I

Time : 3 Hours] **[Max. Marks : 200**

*Answer any **five** questions.*

1. (a) Let $V(K)$ and $U(K)$ be the vector spaces. Let V be a finite dimensional vector space. If $T : V \rightarrow U$ be a linear map, then prove that $dim\ V = dim\ R(T) + dim\ N(T)$,
where $dim\ R(T) =$ Rank of T and $dim\ N(T) =$ Nullity of T.