If they collid, show that the ratio of the initial speeds of A and B is $\dfrac{a\sin\beta + b\cos\beta}{a\sin\alpha + b\cos\alpha}$.

12. (a) Find the centre of pressure of an elliptic area when its major axis is vertical.

(b) A rectangular area is immersed in a heavy liquid with two sides horizontal and is divided by horizontal lines into strips on which the total thrusts are equal. If a, b, c are the breadths of three consecutive strips, then prove that

$$a(a+b)(b-c) = c(b+c)(a-b).$$

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Show that a subgroup N of a group G is normal subgroup of G iff the product of two right co-sets of N in G is again a right co-set of N in G.

(b) Show that a homomorphism ϕ of a group G into a group $\overline{G}$ with Kernel K_ϕ is an isomorphism iff $K_\phi = \{e\}$.

2. (a) Show that every finite group is isomorphic to a permutation group.

(b) Show that a comutative ring with unity is a field if it has no proper ideals.

3. (a) show that in a metric space every convergent sequence is a Cauchy sequence but the converse is not true always.

(b) Let (X, d) be a complete metric space. For each $n \in N$ let F_n be a closed bounded subset of X such that

(i) $F_1 \supset F_2 \supset F_3 \supset \ldots\ldots \supset F_n \supset F_{n+1} \supset \ldots\ldots$ and

(ii) diameter $F_n \to 0$ as $n \to \infty$.

Then show that $\bigcap_{n=1}^{\infty} F^n$ contains precisely one point.

4. (a) Show that improper integral $\int_a^b \frac{dx}{(x-a)^n}$ is convergent when $n < 1$ and divergent when $n \geq 1$.

(b) Show that the function : $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

is not differentiable at (0, 0).

5. (a) Find the maximum value of the function $u = \sin x \sin y \sin(x+y)$.

(b) Show that every absolutely convergent series is convergent but converse is not true.

6. (a) Show that the sum of an absolute convergent series remains unchanged by any rearrangement.

(b) Find the analytic function of which real part is

$$e^{-x}\left\{(x^2-y^2)\cos y + 2xy \sin y\right\}.$$

7. (a) State and prove Cauchy's integral formula.

(b) Find the radius of convergence of power series

$$\sum \frac{n\sqrt{2}+i}{1+2i^n} z^n$$

8. (a) Solve by Charpit's method $px + qy = z(1+pq)^{1/2}$.

(b) Solve : $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2\cos y - x\sin y$.

SECTION–B

9. (a) Deduce the equation of continuity in Lagrangian form.

(b) Deduce Lagrange's stream function.

10. (a) Show that the image of the source m at A consists of an equal sink-m at the centre O of the circle.

(b) Describe irrotational motion in two-dimension.

11. (a) What is Simpson's $\frac{1}{3}$ rule ? Evaluate the integral $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by dividing the interval into 6 points.

(b) Derive the Gaussian quadrature formula for $n = 3$.

12. (a) Find $y(2.2)$ for $\frac{dy}{dx} = xy^2$ where $y(2) = 1$ by Euler's method.

(b) Using Runge-Kutta method find $y(0.1)$ and $y(0.2)$ correct to four decimal of differential equation

$$\frac{dy}{dx} = y - x \text{ with } y(0) = 2.$$

13. (a) The first four moments of a distribution about the value 4 of the variable are -1.5, 17, -30 and 108. Find the moment about the mean and state whether the distribution is leptokurtic or platykurtic.

(b) What is curve fitting ? Fit a second degree parabola to the following :

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

14. (a) Show that the rank correlation coefficient lies between -1 and $+1$ including both the values.

(b) Explain mathematical expectation. Show that the expectation of the sum of two variables is equal to the sum of their expectations.

15. (a) If a coin is tossed N times where N is very large even number, show that the probability of getting exactly $\frac{1}{2}N - p$ heads and $\frac{1}{2}N + p$ tails is approximately $\left(\frac{2}{\pi N}\right)^{1/2} e^{-2p^2/N}$.

(b) Using simplex method, find the inverse of the following matrix :

$$A = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}.$$

16. (a) Use dynamic programming to solve the following linear programming problem :

Maximize $Z = 3x_1 + 5x_2$.

Subject to the constraints

$$x_1 \le 4$$
$$x_2 \le 6$$
$$3x_1 + 2x_2 \le 18$$
$$x_1, x_2 \ge 0.$$

(b) Write the Kuhn Tucker conditions and solve the following problem :

Minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

subject to

$$2x_1 + x_2 - x_3 \le 0$$
$$1 - x_1 \le 0$$
$$2 - x_2 \le 0$$
$$-x_3 \le 0.$$