(b) Evaluate : div grad r^n.

(c) Evaluate : $\int_C F.dr$

where $F = x^2y^2i + yj$ and the curve C is $y^2 = 4x$ in the $x-y$ plane from (0, 0) to (4, 4).

11. (a) Show that the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with $\frac{2}{3}$ of the pulley is $a\left[\frac{3}{\log(2+\sqrt{3})}+\frac{4\pi}{3}\right]$.

(b) Two equal uniform rods AB and AC, each of length $2b$ are freely jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ is the angle between them, then $b\sin^3\theta = a\cos\theta$.

12. (a) A particle is projected horizontally with a velocity $\sqrt{\frac{1}{2}ag}$ from the highest point of the outside of a fixed smooth sphere of radius a. Show that it will leave the sphere at a point whose vertical distance below the point of the projection is $\frac{a}{6}$.

(b) A semi-ellipse bounded by its minor axis is just immersed in a liquid the density of which varies as the depth. If the minor axis be in the surface, find the eccentricity in order that the focus may be the centre of pressure.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 100**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) (i) When is a non-empty set G called a Group ?

(ii) Suppose that G is a group such that $(ab)^2 = a^2b^2$ for every $a, b \in G$. Show that G is abelian.

(b) Let $f: G \to G'$ be a homomorphism with kernel K. Then show that K is a subgroup of G and the quotient group G/K is isomorphic to the image of f.

2. (a) What is a ring ? Show that the set $R(x)$ of all polynomials over R from a ring with respect to the usual operations of additions and multiplication of polynomials.

(b) (i) Let f and g be polynomials over a field K with $g \neq 0$. Prove that there polynomials q and r such that $f = qg + r$ where either $r = 0$ or $\deg r < \deg g$.

(ii) Prove that $D = \{a + b\sqrt{2}, a, b \text{ rational}\}$ is a field.

(a) Define a complete metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in a metric space X such that $x_n \to x$ and $y_n \to y$, show that $\rho(x_n, y_n) \to \rho(x, y)$, where $X \times X \to R$.

(b) Let X and Y be metric spaces and f a mapping of X into Y. Then show that f is continuous $\Leftrightarrow f^{-1}(G)$ is open in X whenever G is open in Y.

(a) Show that : $\int_0^\infty x^{a-1}e^{-x}dx$ converges, if and only if, $a > 0$.

(b) Find min $(x^2 + y^2)$ subject to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) If $f(x,y) = \frac{xy}{x^2 + y^2}$, $x^2 + y^2 \neq 0$, $f(0,0) = 0$, examine if $f(x, y)$ is continuous at $(0, 0)$.

(b) Evaluate $\iint \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) dx\, dy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

6. (a) Define an analytic function. Show that the necessary condition for a function $f(z) = u + iv$ to be analytic at any point $z = x + iy$ of the domain D of f is that the four partial derivatives u_x, u_y, v_x, v_y exist and $u_x = v_y$ and $u_y = -v_x$.

(b) Determine the analytic function $f(z) = x + yi$ whose real part X is equal to $\dfrac{2\sin 2x}{e^{2x} + e^{-2y} - 2\cos 2x}$.

7. (a) If $f(z)$ is analytic inside and on the boundary C of a simply connected region R, prove that $f(a) = \dfrac{1}{2\pi z}\oint \dfrac{f(z)}{(z-a)}dz$.

(b) Prove that : $\int_{-\infty}^{\infty} \dfrac{e^{\text{mix}}}{1+x^2}dx = \dfrac{\pi}{2}e^{-m}$.

8. (a) Find the general solution of

$$(y - zx)\,p - (x + yz)\,q = x^2 - y^2 \cdot \left(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right).$$

(b) Solve : $(D^4 - 2D^2 D^{12} + D^{14})\,z = 0, D = \dfrac{\partial'}{\partial x}, D' = \dfrac{\partial}{\partial y}$.

SECTION–B

9. (a) State D' Alembert's principle and deduce the equation of motion of the centre of inertia of a rigid body and the equations of motion relative to the centre of inertia.

(b) Show that the moment of inertia of an ellipse of mass M and semi axes a and b about a tangent is $\dfrac{5}{4}Mp^2$ where p is the perpendicular from the centre on that tangent.

10. (a) Deduce the equation of continuity of a moving fluid in the form $\dfrac{D\rho}{Dt} + \rho\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}\right) = 0$.

$\dfrac{D}{Dt} \equiv \dfrac{\partial}{\partial t} + u\dfrac{\partial}{\partial x} + v\dfrac{\partial}{\partial y} + w\dfrac{\partial}{\partial z}$, the other symbols have the usual meaning.

(b) Find the lines of flow in the two-dimensional fluid motion given by $\phi + i\Psi = -\dfrac{1}{2}n(x+iy)^2 e^{2\,\text{int}}$.

11. (a) Find the missing value of the following table :

x :	2·0	2·1	2·2	2·3	2·4
$f(x)$:	0·136	–	0·112	0·100	0·095

(b) The area A of a circle with diameter d is given for the following values of d :

d :	80	85	90	95	100
A :	5025	5675	6300	7085	7550

Find the approximate value for the area of the circle with diameter 84.

12. (a) Set the Newton-Raphson sequence of iterations for the solution of the equation $f(x) = 0$ in $[a, b] < R$ given $f(x)$ is twice differentiable w.r.t. x within $[a, b]$. State and prove the condition for the convergence of the Newton-Raphson method.

(b) Determine the value of y at the indicated point for the following initial value problem using Euler's method :

$$\frac{dy}{dx} = 2x + y;\ y(0) = 0;\ \text{find } y(0{\cdot}5).$$

Consider five subdivisions.

13. (a) Solve the following linear programming problem :

Max. $Z = 10x + y$

subject to $3x + 4y \le 20{,}000$

$4x + 3y \le 20{,}000$

$x, y \ge 0.$

(b) Solve the following transportation problem :

10	20	5	7	10
13	9	12	8	20
4	15	7	9	30
14	7	1	0	40
3	12	5	19	50
60	60	20	10	

14. (a) Solve the following game by linear programming :

$$\begin{array}{c|ccc} & & B & \\ \hline & -1 & 1 & 1 \\ A & 2 & -2 & 2 \\ & 3 & 3 & -3 \end{array}$$

(b) Solve the quadratic programming problem :

Min $Z = 5x_1 - 2x_1 x_2 + 5x_2^2$

subject to $x_1 + x_2 \leq 1,\ x_1 \geq 0,\ x_2 \geq 0.$

15. (a) State and prove Baye's theorem.

(b) In an examination marks obtained by students in Mathematics Physics and Chemistry are distributed normally, about th means 50, 52, 48 with *s.d.* 15, 12, 16 respectively. Find the probability of scoring total marks of 180 or above.

$$\left[\frac{1}{\sqrt{2\pi}}\int_{1\cdot2}^{\infty} e \times p\left(-\frac{t^2}{2}\right) dt = 0\cdot1942\right]$$

16. (a) Fit a straight line of the form $y = a + bx$ to the following da by the method of least squares :

$x =$	1	2	3	4	5	6	7	8
$y =$	38	46	50	51	55	56	58	59

(b) If X is a Poissonian variate with parameter μ, show th the distribution $\frac{(X-\mu)}{\sqrt{\mu}}$ tends to that of a normal variate a $\mu \to +\infty$.