MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Show that a non-empty subset H of a group G is a subgroup of G if $a, b \in H \Rightarrow ab^{-1} \in H$.
   (b) Show that every homomorphic image of a Group G is isomorphic to some quotient group of G.
2. (a) If R is a commutative ring with unit element whose only ideals are (0) and R itself. Then show that R is a field.
   (b) Show that if R is unique factorization domain then so is $R[x]$.
3. (a) Define complete metric space. Show that if A is a closed subset of a complete metric space (X, d) and $A \subset X$, then (A, d) is also complete.
   (b) Let (M_1, d_1) be a complete metric space. If f is continuous function from M_1 into a metric space (M_2, d_2), then show that f is uniformly continuous on M_1.
4. (a) Test for the convergence of improper integral

$$\int_0^{\infty} \frac{dx}{x(\log x)^{n+1}} (a > 1)$$

   (b) Find maxima and minima of $u = \sin x \sin y \sin z$ where x, y and z are vertical angles of a triangle.
5. (a) Show that the function :

$$f(x, y) = \frac{xy^2}{x^2 + y^2}; \quad (x, y) \neq (0, 0)$$
$$= 0 \qquad \text{otherwise}$$

   is not differential at (0, 0).

   (b) Show that the sequence $\{f_n(x)\}_{n=1}^{\infty}$ where $f_n(x) = nx(1-x)^n$ does not converge uniformly on [0, 1].
6. (a) Define analytic function. Show that necessary condition for a function $f(z) = u + iv$ to be analytic at any point $z = x + iy$ of the domain D of f is that the four partial derivatives u_x, u_y, v_x, v_y exist and $u_x = v_y$ and $u_y = -v_x$.

Discuss the suggestion that the mean height in the universe is 65 inches, given that for 9 degree of freedom the value of Student's t at 5% level of significance is 2·262.

16. (a) What is curve fitting ? Fit a second degree parabola to the following :

x :	0	1	2	3	4
y :	1	1·8	1·3	2·5	6·3

(b) Define partial and multiple correlation for a trivariate distribution. Show that :

(i) $$R_{1(23)}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}}{1 - r_{23}^2}$$

(ii) $$1 - R_{1(23)}^2 = (1 - r_{12}^2)(1 - r_{13.2}^2)$$