(b) Prove that every group is isomorphic to a subgroup of $A(S)$ for some appropriate S.

2. (a) Let J be the ring of integers, J_n, the ring of integers modulo n. Define $\phi : J \to J_n$ by $\phi(a)$ = remainder of a on division by n. Verify that ϕ a homomorphism of J on to J_n.

(b) If U is an ideal of the ring R, then show that R/U is a ring and is a homomorphic image of R.

3. (a) Prove that the function $f(x) = |x|$ is continuous at $x = 0$ but is not differentiable at $x = 0$.

(b) State Malaurin's theorem. Expand $\sin x$ as a power series in x.

4. (a) State Cauchy's principle of convergence

Prove that $\lim\limits_{x \to \infty}\left(1+\dfrac{1}{n}\right)^n = e$.

(b) Given $f(x) = \dfrac{1}{4}\pi x, 0 \le x \le \dfrac{1}{2}\pi$

$$f(x) = \frac{1}{4}\pi(\pi - x), \frac{1}{2}\pi \le x \le \pi.$$

find a series of cosines of multiples of x which will represent $f(x)$ in the interval $0 \le x \le \pi$.

5. (a) Show that the improper integral,

$\displaystyle\int_0^b \frac{dx}{(x-a)^\mu}$ converges if and only if $\mu < 1$.

(b) If $\dfrac{x^2}{a^2+u} + \dfrac{y^2}{b^2+u} + \dfrac{z^2}{c^2+u} = 1$

where u is a function of x, y, z prove that

$$(u_x)^2 + (u_y)^2 + (u_z)^2 = 2(xu_x + yu_y + zu_z).$$

6. (a) Determine the analytic function $f(z) = X + iY$, whose real part X is equal to $\dfrac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$.

(b) Prove the formula $\cos a + \cos(a+b) + \ldots.. + \cos(a+nb)$

$$= \frac{\sin\left(\frac{n+1}{2}b\right)}{\sin\left(\frac{b}{2}\right)}\cos\left(a+\frac{nb}{2}\right).$$

7\. (a) Let $f(z)$ be an analytic function in the finite region A limited by a boundary Γ composed of one or of several distinct closed curves and continuous on the boundary itself.

If x is a point of the region A, the function $\frac{f(z)}{z-x}$ is analytic in the same regoin except at the point $x = a$. Prove this.

(b) Prove that $\int_{-\infty}^{+\infty} \frac{e^{mix}}{1+x^2}dx = \pi e^{-m}$.

8\. (a) Solve : $x(y^2-1)\,dx + y(x^2-1)\,dy = 0$.

(b) Solve : $(D^2 - D') = z = 2y - x^2$ where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$.

SECTION–B

9\. (a) State D' Alembert's principle and deduce the equations of motion of the centre of inertia of a rigid body and the equations of motion relative to the centre of inertia.

(b) Show that a uniform rod of mass M is equimomental to three particles each of mass $\frac{1}{5}M$ placed at each end of the rod and a mass $\frac{2}{3}M$ placed at its middle point.

10\. (a) A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is $\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial\theta}(\rho\omega) = 0$.

(b) Let u, v, w to be the components of velocity, ρ the density and ρ the pressure at the point (x, y, z) increase of fluid, and let x, y, z be the components of external forces per unit mass at the same point. Deduce Euler's Dynamical equations.

11. (a) Solve the following linear programming problem by simplex method :

Max. $Z = 5x_1 - 2x_2 + 3x_3$

subject to $2x_1 + 2x_2 - x_3 \geq 2;$ $3x_1 - 4x_2 \leq 3;$

$x_2 + 3x_3 \leq 5;$ $x_1, x_2 \geq 0.$

(b) Prove that the dual of the dual is the primal. Verify the theorem for the LPP :

Max. $Z = 3x_1 + 4x_2$

subject to $2x_1 + x_2 \leq 5;$ $x_1 + x_2 \leq 3;$ $x_1, x_2 \geq 0.$

12. (a) Prove that the value of a two-person zero-sum game is unique.

(b) Solve the game with the following pay-off matrix.

$$\begin{pmatrix} 0 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{pmatrix}$$

13. (a) What is a transportation problem ? Solve the following transportation problem :

	A	B	C	D	
I	10	20	5	7	15
II	18	9	12	8	25
III	15	14	16	18	5
	5	15	15	10	

(b) Find the extreme points of introduction

$$f(x_1, x_3) = x_1^3 x_2^3 + 2x_1^2 + 4x_2^2 + 6x_1x_2 + 7x_1x_3 + 9x_2x_3 + 13$$

14. (a) Complete the following table by supplying values of $f(x)$ which are missing :

x :	1	2	3	4	5	6
$f(x)$:	2	10	–	–	130	222

(b) What is Simpson's $\frac{1}{3}$rd rule ? Compute the integral $\int_0^1 e^{x^2} dx$ by Simpson's $\frac{1}{3}$rd rule for $n = 10$.

15. (a) What is Newton-Raphson method for the solution of the equation $f(x) = 0$? Present line geometric interpretation of the method. Prove that the rate of convergence is quadratic under conditions to be stated by you.

(b) Solve the differential equation $\frac{dy}{dx} = xy^2, y = 2, x = 0$ by Runge-Kutta method or otherwise taking $h = 0.1$ and obtain y at $x = 0.2$.

16. (a) An urn contains a white and b black balls and a series of drawings of one ball at a time is made. The ball removed being returned to the urn immediately after the next drawing is made. If p_n denotes the probability that the n^{th} ball drawn is black, show that $p_n = (b - p_n - 1)/(a + b - 1)$.

(b) Define coefficient of variation. What purpose does it serve ? Distinguish between variance and coefficient of variation. The runs scored by two batsmen A and B in 8 innings are as follows :

A:	10	115	5	73	7	120	36	84
B:	45	12	76	42	4	50	37	48

Find which batsman is more consistent.