velocity v; if $\frac{m}{M}$ be small, show that the major axis of the planet's path is reduced by $\frac{4m}{M}\frac{VT}{\pi}\cdot\sqrt{\frac{1-e}{1+e}}$.

(b) A particle falls from rest at a distance a from the centre of the Earth towards the Earth, the motion meeting with small resistance proportional to the square of the velocity v and the retardation being μ for unit velocity. Show that the kinetic energy at distance x from the centre is

$$mgr^2\left\{\frac{1}{x}-\frac{1}{a}+2\mu\left(1-\frac{x}{a}\right)-2\mu\log\frac{a}{x}\right\},$$

the square of μ being neglected and r being the radius of the Earth.

16. (a) A closed tube in the form of an ellipse, with its major axis vertical, is filled with three different liquids of densities ρ_1, ρ_2 and ρ_3 respectively. If the distances of the surfaces of separation from either focus be γ_1, γ_2 and γ_3 respectively, prove that

$$\gamma_1(\rho_2-\rho_3)+\gamma_2(\rho_3-\rho_1)+\gamma_3(\rho_1-\rho_2)=0.$$

(b) A trapezium $ABCD$ having AD, BC as its parallel sides, is immersed in a liquid with its plane vertical. If the corners A, B, C and D are at depths α, β, γ and δ respectively, find the depth of the centre of pressure of the trapezium.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) If f is a homomorphism of a group G on to a group G' and if the order of $a \in G$ is finite say n then show that the order of $f(a)$ i.e. m is a divisor of n.

(b) Show that the intersection of any two normal subgroups of a group is a normal subgroup.

2. (a) Prove that the set of all real numbers of the form $m+n\sqrt{2}$, where m and n are integers with ordinary addition and multiplication forms a ring.

(b) Define principal ideal domain and show that every principal ideal domain is a unique factorisation domain.

3. (a) Prove that the function f defined as follows :

$$f(x) = \begin{cases} x \sin\dfrac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$$ is continuous at $x = 0$.

(b) Prove that the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^3}$ is uniformly convergent for all values of x and that it may be differentiated term by term.

4. (a) Define Cauchy sequence and show that every Cauchy sequence is bounded. Also prove that the sequence $\left\{\frac{1}{n}\right\}$ is a Cauchy's sequence.

(b) Show that $\int_0^{\infty} \frac{\sin x}{x} dx$ is convergent but not absolutely convergent.

5. (a) Evaluate : $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z} dz\, dy\, dx$.

(b) Show that : $U = x^3y^2(1 - x - Y)$ is maximum at $x = \frac{1}{2}, y = \frac{1}{3}$.

6. (a) State and prove Cauchy's theorem.

(b) By using Taylor's series, prove that

$$\log z = (z-1) - \frac{(z-1)^2}{2} + \ldots.$$

7. (a) State and prove Cauchy's residue theorem.

(b) By Contour integration show that : $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$.

8. (a) Solve : $(z^2 - 2yz - y^2)\, p + (xy + xz)\, q = xy - xz$.

(b) Solve the differential equation :
$(D^3 - 4D^2D' + 4DD'^2)\, z = 4 \sin(2x + y)$,
where $D = \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$.

SECTION–B

9. (a) Obtain Lagrange's equations of motion in generalised co-ordinates.

(b) Find the moment of inertia of a rectangular lamina about one of its sides. State and prove D' Alembert's principle.

10. (a) Show that : $u = \frac{3x^2 - r^2}{r^5}, v = \frac{3xy}{r^5}, w = \frac{3xz}{r^5}$,

where $r^2 = x^2 + y^2 + z^2$ are the velocity components of a possible liquid motion. Is this motion irrotational ?

(b) Obtain the equation of continuity in the following form :

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{u}) = 0$$

11. (a) Solve the following linear programming problem by simplex method : Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints

$2x_1 + 3x_2 \leq 8 \qquad 3x_1 + 2x_2 + 4x_3 \leq 15,$

$2x_2 + 5x_3 \leq 10$ and $x_1, x_2, x_3 \geq 0$

(b) Show that if either primal or the dual problem has a finite optimal solution, then the other problem has also a finite optimal solution. Also show that the optimal values of the objective functions in both the problems are the same.

12. (a) Discuss Bellman's optimality principle.

(b) Five jobs are to be assigned to 4 machines subject to the cost matrix as shown below. Make a minimum cost assignment :

JOB	MACHINE			
	A	B	C	D
I	9	7	6	2
II	6	6	7	6
III	5	3	4	4
IV	4	2	5	9
V	2	8	3	9

13. (a) A company has a factories at A, B and C which supply warehouses at D, E, F and G. Unit shipping costs (in Rs.) are given in the table. Determine the optimum distribution for this company to minimise shipping costs.

From	To				Capacity
	D	E	F	G	
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
Demand	80	90	110	160	

(b) Discuss the steady state and transient solutions for queuing system with Position arrivals and exponential service time.

14. (a) Find the polynomial of the lowest possible degree which assumes the values 3, 12, 15, –21; when x has values 3, 2, 1, –1 respectively.

(b) Find the real root of the equation $x^3 - 3x - 5 = 0$ correct to three decimal places by Newton-Raphson method.

15. (a) Use Runge's method to approximate y when $x = 0.1$ given that

$$y = 1 \text{ at } x = 0 \text{ and } \frac{dy}{dx} = x + y.$$

(b) Assuming Stirling's interpolation formula, prove that :

$$\frac{d}{dx}(y_x) = \frac{2}{3}[y_{x+1} - y_{x-1}] - \frac{1}{12}[Y_{x+12} - Y_{x-2}]$$

Considering the differences upto third order.

16. (a) A bag contains 7 black, 12 white and 4 green balls. Find the probability that :

(i) a ball selected at random is white

(ii) Three balls selected are white

(iii) Three balls selected are of different colours.

(b) Discuss briefly regression, correlation and correlation ratio. Discuss also rank correlation, partial correlation coefficient and multiple correlation coefficient.