constants. Show that the motion is Simple Harmonic if c is positive and determine the period in terms of a, b and c.

(c) Of a point moves in a plane such that the velocities parallel to the axes of x and y are $u + ay$ and $v + bx$ respectively, show that is moves in a conic section.

12. (a) Obtain the fundamental pressure equation for the equilibrium of a finite mass of fluid and hence obtain the necessary and sufficient conditions of equilibrium.

(b) A hemisphere is filled with homogeneous liquid. Find the resultant action on one of the four portions into which it is divided by two vertical planes through the centre at right angles to each other.

(c) Two volumes V, V' of different gases, at pressure p, p' respectively are mixed together, so that the volume of the mixture is U. Find the pressure of the mixtue.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) If H is a subgroup of G, show that for every $g \in G$, $H g^{-1}$ is subgroup of G.

(b) If p is the smallest prime factor of the order of a finite group G. Prove that any subgroup of index p is normal.

(c) Show that every maximal ideal of a commutative ring R with unit element must be a prime ideal.

2. (a) Show that every irrducible element of a principal ideal domain is prime.

(b) Let F be a field having q element Prove that

(i) Characteristic of F is p for some prime number p.

(ii) $q = p^n$ for some $n \in N$.

(c) The field k is an extension of field F. If a, b in k are both algebraic over F, show that $a + b$, ab and $\frac{a}{b}(b \neq 0)$ are all algebraic over E.

3. (a) Let X and Y be metric spaces and f a mapping of X into Y. Show that f is continuous on X if, and only if the reverse immage of open sets in Y under f is open in X.

(b) Show that a Cauchy sequence in a metric space is convergent if, and only if it has a convergent subsequence.

(c) Show that a continuous mapping of a compact metric space E into a metric E is uniformly continuous.

4. (a) Find the maximum value of xyz where $x + y + z = a$.

(b) Show that $\int_0^{\pi/2} x^m \operatorname{cosec}^n x \, dx$

exists if, and only if, $n < m + 1$.

(c) Prove that the function $f(x)$ defined by

$$f(x) = \frac{1}{2^n} \phi \frac{2}{2^m} - < x \leq \frac{1}{2^n} (n = 0, 1, 2 \ldots.)$$

$f(0) = 0$ is integrable in the closed interval $[0, 1]$ even through it is not continuous in $[0, 1]$.

5. (a) What do you mean by saying that an infinite product $\prod_1^{\infty} Un$ is convergent with product p?

Show that the infinite product $\prod_1^{\infty}\left(1+\frac{1}{n^2}\right)$ is convergent.

(b) If $U_n = \frac{1+nx}{ne^{n+}} - \frac{1+(n+1)x}{(n+1)e^{(n+1)}x}, 0 < x < 1$

Prove that $\frac{d}{d^n} \sum_1^{\infty} U^n = \sum_1^{\infty} . \frac{d}{d^n} U^n$.

Is the series, $\Sigma U^n(x)$ uniformly convergent in $(0, 1)$? Give reasons for your answer.

(c) Evaluate $\iint \frac{\sqrt{(a^2b^2 - bx^2 - a^2y^2)}}{\sqrt{(a^2b^2 + b^2x^2 + a^2y^2)}} dx\, dy$

the region of integration being the positive quadrant of

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

6. (a) If the harmonic functions $u(x, y)$ and $v(x, y)$ satisfy Candry Riemann differential equations, then show that $u + iv$ an analytic function.

(b) Prove that an analytic function in a region whose real and imaginary parts u and v satisfy the equation $v - u^2$ must reduce to a constant.

(c) State and prove Cauchys theorean.

7. (a) Show that a function which is analytic in the whole plane and has a non-essential singularity at ∞ reduces to a polynormal.

(b) Using the method of residues, show that $\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx - \frac{\pi}{e}$.

(c) Find the complete integral of the equation

$$px + qy = z(1 + pq)^{1/2}.$$

8. (a) Solve : $(3x + y - z)p + (x + y - z)q = 2(z - y)$.

(b) Solve : (i) $\frac{\delta^2 Z}{\delta x^2} - \frac{\delta^2 Z}{\delta x \delta xy} - 2\frac{\delta Z}{\delta y^2} = (y-1)eX$

(ii) $\frac{\delta^2 Z}{\delta x^2} + 5\frac{\delta^2 Z}{\delta x \delta y} + \frac{\delta^2 Z}{\delta y^2} = \frac{1}{y-2x}$.

SECTION–B

9. (a) Give examples of mechanical systems with 4, 5 and 7 degrees of freedom. Explain the distinction between holonomic and non-holonomic constraints with illustrative examples.

(b) When the hegrangian function has the form

$$L = q_k q_k - \sqrt{(1 - q_k^2)},$$

show that the generalized acceleration is Zero.

(c) A uniform triangular lamina ABC can oscillate in its own plane about the vertex A. Prove that the length of the equivalent simple pendulam is $\frac{3(b^2 + c^2) - a^2}{4\sqrt{2b^2 + 2c^2 - a^2}}$, the axis of rotation being horizontal.

10. (a) An infinite mass of fluid is acted on by a force $\frac{\mu}{1^{3/2}}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of a sphere $\gamma = c$ in it. Show that cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{1/2/5/4}$.

(b) A pulse travelling along a fine straight uniform tube filled with gas causes the density at time t and distance x from the origin where the velocity is U_0 to become $1_0\phi(vt - x)$. Prove that the velocity U (at time t and distance x from the origin) is given by

$$v + \frac{(U_0 - v)\,\phi cvt)}{\phi(vt - x)}.$$

11. (a) Explain how to find the motion due to a two dimentional source in the presence of a circular boundary.

A Source S and a snik T of equal strength m are situated within the space bounded by a circle whose centre is O. If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB, show that the velocity of the liquid at any point P is

$$2m \cdot \frac{OS^2 + OA^2}{OS} \cdot \frac{PA \cdot PB}{PS \cdot PS' \cdot PT \cdot PT'}$$

where S' and T' are the universe points of S and T with respect to the circle.

(b) Between the fixed boundaries $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{6}$, their is a two-dimensional liquid motion due to a source at the point $(\gamma = c, \theta = \alpha)$, and a snik at the origin absorbing water at the same rate as the source produces it. Find the stream function and show that one of the stream lines is a part of the curve $\gamma^2 \sin 3\alpha = c^3 \sin 3\theta$.

12. (a) Using Regula-Falsi method, find the real root of the equation $x \log_{10} x - 1{,}2 = 0$, correct to 5 decimal places.

(b) Fit the natural cubic spline for the data

$x:$	0	1	2	3	4
$y:$	0	0	1	0	0

(c) Given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0)$ 1,

$y(0.1)$ ... 1.0954.$y(0.2)$ ... 1.1832 and y (0.3) ..., 1.2694 find y (0.5) by Adam's Predictor-Corrector method.

13. (a) Show that, for any discrete distribution, the standard deviation is not less than the mean deviation from the mean.

(b) Each of the nurns contain a white and b black balls. One ball is transformed from the first urn into the second, then one ball from the second into the third and so on. Finally one ball is taken from the last urn. What is the probability of its being white ?

(c) Two discrete random variable X and Y have the joint probability density function.

$$p(x, y) = \frac{\lambda^x e^- p^p (1-p)^{x-p}}{y!(x-y)!}, = 0, 1 \ldots\ldots x,$$

$$x = 0, 1, 2 \ldots\ldots$$

where $\lambda \cdot p$ are constant and $0 < p < 1 : \lambda > 0$.

Find, (i) the marginal probability density functions of X and Y.

(ii) The conditional distributions of Y for a given X.

14. (a) Fit a second degree parabola to the following data

$x:$	1	2	3	4	5	6	7	8	9
$y:$	2	6	7	8	10	11	11	10	0

(b) For a random sample of 10 horses, fed on diet A, the increase in weight in kilograms in a certain period were

10, 6, 16, 17, 13, 21, 8, 14, 15, 9.

For another random sample of 12 horses fed on diet

7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 kgs.

Test whether the diets A and B differ significantly as regards the effect on increase in weight. You may use the fact that 5% value of t for 20 degrees of freedom is 2.09.

(c) Let $X_1^2\ X_2^2$ be independently distributed variates having chi-square distributions with $n_1 n_2$ degrees of freedom respectively.

Derive the distribution of $F = \dfrac{X_1^2/n_1}{X_2^2/n^2}$.

15. (a) Compute all the basic feasible solutions of the linear programming problem

Maximize $z = 5x_1 + 6x_2 - 7x_3 + 9x_4$

Subject to
$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 + x_4 &= 9 \\ x_1 + 2x_2 + 4x_3 + 3x_4 &= 2 \\ 3x_1 + x_2 + x_3 - 2x_4 &= 5 \\ x_1, x_2, x_3, x_4 &\le 0 \end{aligned}$$

and choose that one which maximizes z.

(b) Find the optional strategies for two players of the rectangular game whose pay-off matrix is $\begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$.

(c) A company has 4 machines which are required to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table :

		W	X	Y	Z
Job	A	18	24	28	2
	B	8	13	17	18
	C	10	15	19	22

What are the job-assignments which will minimize the cost ?

16. (a) Consider a self service store with one cashier. Assume Poisson arrivals and exponential service times. Suppose that 9 customers arrive on the average every 5 minutes and the cashier can serve 10 in 5 minutes. Find

(i) The average number of customer queueing for service ?

(ii) The probability that a customer has to queue for more than 2 minutes ?

(b) Find the sequence that minimizes the total elapsed time required to complete the following tasks. Each job is processed in the order ABC.

		Job					
		1	2	3	4	5	6
Machine	A	18	12	29	30	43	37
	B	7	12	11	2	6	12
	C	19	12	23	47	28	36

What should be the sequence of the jobs ?

(c) State Bellman's Principle of Optimality in dynamic programming and use it to solve

Minimize $x_1 \, x_2 \, ... \, x_3$

Subject to $x_1 + x_2 + ... + x_n = c; \quad x_1 \, x_2 \, ... \geq x_n \; 0.$