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UPSC MATHS OPTIONAL COACHING

Vector Analysis

Previous year Questions from 2020 to 1992

2021-22

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2020

1. For what value of a,b,c is the vector field $\overline{V}=(-4x-3y+az)i+(bx+3y+5z)j+(4x+cy+3z)k$ irrotational? Hence, express $\overline{V}$ as the gradient of a scalar function ϕ determine ϕ **[10 Marks]**
2. For the vector function $\overline{A}$ where $\overline{A}=(3x^2+6y)i-14yzj+20xz^2k$, calculate $\int_C \overline{A}.d\overline{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following paths:
(i) $x=t, y=t^2, z=t^3$
(ii) Straight lines joining $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$
(iii) Straight line joining $(0,0,0)$ to $(1,1,1)$ is the result same in all the cases? Explain the reason. **[15 Marks]**
3. Verify the stokes theorem for the vector field $\overline{F}=xyi+yzj+xzk$ on the surface S which is the part of the cylinder $z=1-x^2$ for $0\le x\le 1, -2\le y\le 2;$ S is oriented upwards. **[20 Marks]**
4. Evaluate the surface integral $\iint_s \nabla\times\overline{F}.nds$ for $\overline{F}=yi+(x-2xz)j-xyk$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy-plane **[15 Marks]**

2019

5. Find the directional derivative of the function $xy^2+yz^2+zx^2$ along the tangent to the curve $x=t^2, y=t^2, z=t^3$ at the point $(1,1,1)$ **[10 Marks]**
6. Find the circulation of $\vec{F}$ round the curve C where $\vec{F}=(2x+y^2)i(3y-4x)j$ and C is the curve $y^2=x$ from $(0,0)$ to $(1,1)$ and the curve $y=x^2$ from $(1,1)$ to **[15 Marks]**
7. Find the radius of curvature and radius of torsion of the helix $x=a\cos u,\ \ y=a\sin u, z=au\tan\alpha$ **[15 Marks]**
8. State Gauss divergence theorem. Verify this theorem for $\vec{F}=4xi-2y^2j+z^2k$ taken over the region bounded by $x^2+y^2=4, z=0$ and $z=3$ **[15 Marks]**
9. Evaluation by Stoke's theorem $\oint_C e^x dx+2ydy-dz$ where C is the curve $x^2+y^2=4,\ \ z=2$. **[05 Marks]**

2018

10. Find the angle between the tangent at a general point of the curve whose equations are $x=3t, y=3t^2, z=3t^3$ and the line $y=z-x=0$ **[10 Marks]**
11. Let $\vec{v}=v_1\vec{i}+v_2\vec{j}+v_3\vec{k}$ .Show that curl (curl $\vec{v}$) =grad (div $\vec{v}$) $-\nabla^2\vec{v}$. **[12 Marks]**
12. Evaluate the line integral $\int_C -y^3dx+x^3dy+z^3dz$ using stokes theorem. Here C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x+y+z=1$. The orientation on C corresponds to counterclockwise motion in the xy-plane. **[13 Marks]**
13. Let $\vec{F}=xy^2\vec{i}+(y+x)\vec{j}$ Integrate $(\nabla\times\vec{F}).\vec{k}$ over the region in the first quadrant bounded by the curves $y=x^2$ and $y=x$ using Green's theorem. **[13 Marks]**
14. Find the curvature and torsion of the curve $\vec{r}=a(u-\sin u)\vec{i}+a(1-\cos u)\vec{j}+bu\vec{k}$ **[12 Marks]**

15. If S is the surface of the sphere $x^2+y^2+z^2=a^2$, then evaluate $\iint_S \left[(x+z)\,dydz+(y+z)dzdx+(x+y)dxdy\right]$ using gauss' divergence theorem. **[12 Marks]**

2017

16. For what values of the constant a, b and c the vector $\bar{V}=(x+y+az)\hat{i}+(bx+2y-z)j+(-x+cy+2z)k$ is irrotational. Find the divergence in cylindrical coordinates of the vector with these values. **[10 Marks]**

17. The position vector of a moving point at time t is $\bar{r}=\sin t\ \hat{i}+\cos 2t\ \ j+(t^2+2t)\ k$. Find the components of acceleration $\bar{a}$ in the direction parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time $t=0$. **[10 Marks]**

18. Find the curvature vector and its magnitude at any point $\bar{r}=(\theta)$ of the curve $\bar{r}=(a\cos\theta, a\sin\theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2+y^2-z^2=a^2$. **[16 Marks]**

19. Evaluate the integral $\iint_S \vec{F}.nds$ where $\vec{F}=3xy^2\hat{i}+(yx^2-y^3)j+3zx^2K$ and S is a surface of the cylinder $y^2+z^2\le 4,\ -3\le x\le 3$ using divergence theorem. **[9 Marks]**

20. Using Green theorem evaluate the $\int_C F(\vec{r}).d\vec{r}$ counterclockwise where $F(\vec{r})=(x^2+y^2)\hat{i}+(x^2-y^2)j$ and $d\vec{r}=x\hat{i}+dy\,j$ and the curve C is the boundary off the region $R=\left\{(x,y)\middle|\ 1\le y\le 2-x^2\right\}$. **[8 Marks]**

2016

21. Prove that the vector $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}, \vec{b}=-\hat{i}+3\hat{j}+4\hat{k}, \vec{c}=4\hat{i}-2\hat{j}-6\hat{k}$ can from the sides of a triangle find the length of the medians of the triangle **[10 marks]**

22. Find $f(r)$ such that $\nabla f=\dfrac{\vec{r}}{r^5}$ and $f(1)=0$ **[10 marks]**

23. Prove that $\oint_C fd\vec{r}=\iint_S d\vec{S}\times\nabla f$ **[10 marks]**

24. For the of cardioid $r=a(1+\cos\theta)$ show that the square of the radius of curvature at any point (r,θ) is proportion to r. Also find the radius of curvature if $\theta=0,\dfrac{\pi}{4},\dfrac{\pi}{2}$. **[15 marks]**

2015

25. Find the angle between the surfaces $x^2+y^2+z^2-9=0$ and $z=x^2+y^2-3$ at $(2,-1,2)$ **[10 Marks]**

26. A vector field is given by $\vec{F}=(x^2+xy^2)\hat{i}+(y^2+x^2y)\hat{j}$. Verify that the field is irrotational or not. Find the scalar potential. **[12 Marks]**

27. Evaluate $\int_C e^{-x}(\sin y dx+\cos y dy)$, where C is the rectangle with vertices $(0,0)(\pi,0),\left(\pi,\dfrac{\pi}{2}\right),\left(0,\dfrac{\pi}{2}\right)$ **[12 Marks]**

2014

28. Find the curvature vector at any point of the curve $\bar{r}(t) = t\cos t\hat{i} + t\sin t\hat{j},\ 0 \le t \le 2\pi$. Give its magnitude also. **[10 Marks]**

29. Evaluate by Stokes' theorem $\int_{\Gamma}\left(ydx + zdy + xdz\right)$, where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y = 2a$ starting from $(2a,0,0)$ and then going below the z- plane. **(20 Marks]**

2013

30. Show the curve $\vec{x}(t) = t\hat{i} + \left(\dfrac{1+t}{t}\right)\hat{j} + \left(\dfrac{1-t^2}{t}\right)\hat{k}$ lies in a plane. **[10 Marks]**

31. Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x,y,z) from the origin, n being a constant and ∇^2 being the Laplace operator **[10 Marks]**

32. A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$ **[10 Marks]**

33. By using Divergence Theorem of Gauss, evaluate the surface integral $\iint\left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-\frac{1}{2}}\,dS$, where S is the surface e of the ellipsoid $ax^2 + by^2 + cz^2 = 1,\ a,b$ and c being all positive constants. **[15 Marks]**

34. Use Stokes' theorem to evaluate the line integral $\int_C(-y^3dx + x^3dy - z^3dz)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$ **[15 Marks]**

2012

35. If $\vec{A} = x^2yz\vec{i} - 2xz^3\vec{j} + xz^2\vec{k},\ \vec{B} = 2z\vec{i} + y\vec{j} - x^2\vec{k}$ find the value of $\dfrac{\partial^2}{\partial x\partial y}(\vec{A} + \vec{B})$ at $(1,0,-2)$ **[12 Marks]**

36. Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve $x = t, y = t^2, z = \dfrac{2}{3}t^3$. Show that the curvature and torsion are equal for this curve. **[20 Marks]**

37. Verify Green's theorem in the plane for $\oint_C\left[xy + y^2dx + x^2dy\right]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ **[20 Marks]**

38. If $\vec{F} = y\vec{i} + \left(x - 2xz\right)\vec{j} - xy\vec{k}$, , evaluate $\iint_S(\vec{\nabla} \times \vec{F}).\vec{n}d\vec{s}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy- plane. **[20 Marks]**

2011

39. For two vectors $\vec{a}$ and $\vec{b}$ give respectively by $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin 5t\hat{i} - \cos t\hat{j}$ determine:(i) $\frac{d}{dt}\left(\vec{a}.\vec{b}\right)$ and (ii) $\frac{d}{dt}\left(\vec{a}\times\vec{b}\right)$ **[10 Marks]**

40. If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f} = grad v,$ find the value of $\vec{f} curl \vec{f}$ **[10 Marks]**

41. Examine whether the vectors $\nabla u, \nabla u$ and ∇w are coplanar, where u, v and w are the scalar functions defined by:

$$u = x + y + z,$$
$$v = x^2 + y^2 + z^2$$
$$\text{and } w = yz + zx + xy$$

[15 Marks]

42. If $\vec{u} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$ calculate the double integral $\iint\left(\nabla\times\vec{u}\right)d\vec{s}$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2, z \geq 0$ **[15 Marks]**

43. If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector. $r^n\vec{r}$ is (i) irrotational, (ii) solenoidal **[15 Marks]**

44. Verify Gauss' Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube $0 \leq x, y, z \leq 1.$ **[15 Marks]**

2010

45. Find the directional derivative of $f\left(x, y\right) = x^2y^3 + xy$ at the point (2,1) in the direction of a unit vector which makes an angle or $\frac{\pi}{3}$ with the x-axis. **[12 Marks]**

46. Show that the vector field defined by the vector function $\vec{v} = xyz\left(yz\vec{i} + xy\vec{j} + xy\vec{k}\right)$ is conservative. **[12 Marks]**

47. Prove that $div\left(f\vec{V}\right) = f\left(div\vec{V}\right) + \left(grad.f\right)\vec{V}$ where f is a scalar function. **[20 Marks]**

48. Use the divergence theorem to evaluate $\iint_s \vec{V}ndA$ where $\vec{V} = x^2z\vec{i}y\vec{j} - xz^2\vec{k}$ and S is he boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$. **[20 Marks]**

49. Verify Green's theorem for $e^{-x}\sin y dx + e^{-x}\cos y$ by the path of integration being the boundary of the square whose vertices are $\left(0,0\right), \left(\frac{\pi}{2,0}\right)\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\left(\frac{0,\pi}{2}\right)$ **[20 Marks]**

2009

50. Show that $div\left(grad r^n\right) = n\left(n+1\right)r^{n-2}$ where $r = \sqrt{x^2 + y^2 + z^2}$. **[12 Marks]**

51. Find the directional derivative of $(i) 4xz^3 - 3x^2y^2z^2$ (i) at $\left(2,-1,2\right)$ along z-axis $(ii) - x^2yz + 4xz^2$ at $\left(1,-2,1\right)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{\kappa}$. **[6+6=12 Marks]**

52. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z=0$ under the field of force of given by $\overline{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}(3x-2y+4z)\hat{k}$. **[20 Marks]**

53. Using divergence theorem, evaluate $\iint_s \overline{A}.d\overline{S}$ where $\overline{A}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$ **[20 Marks]**

54. Find the value of $\iint_s (\vec{\nabla}\times\vec{f}).d\vec{s}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane $z=0$,when $\vec{F}=(y^2+z^2-x^2)\hat{i}+(z^2+x^2-y^2)\hat{j}+(x^2+y^2-z^2)\hat{k}$ **[20 Marks]**

2008

55. Find the constants a and b so that the surface $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point $(1,-1,2)$. **[12 Marks]**

56. Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from $(1-2,1)$ to $(3,1,4)$. **[12 Marks]**

57. Prove that $\nabla^2 f(x)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$ where $r=(x^2+y^2+z^2)^{\frac{1}{2}}$. Hence find $f(x)$ such that $\nabla^2 f(r)=0$. **[15 Marks]**

58. Show that for the space curve $x=t, y=t^2, z=\frac{2}{3}t^3$ the curvature and torsion are same at every point. **[15 Marks]**

59. Evaluate $\int_c \vec{A}d\vec{r}$ along the curve $x^2+y^2=1, z=1$ form $(0,1,1)$ to $(1,0,1)$ if $\vec{A}=(yz+2x)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}$. **[15 Marks]**

60. Evaluate $\iint_s \vec{F}\hat{n}d$ were $\vec{A}=(yz+2x)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}, \iint_s \vec{F}\hat{n}ds$ and S is the surface of the cylinder bounded by $x^2+y^2=4z=0$ and $z=3$ **[15 Marks]**

2007

61. If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r=|\vec{r}|$ determined grad(r_{-1}) in terms of $\hat{r}$ and r. **[12 Marks]**

62. Find the curvature and torsion at any point of the curve $x=a\cos 2t, y=a\sin 2t, z=2a\sin t$. **[12 Marks]**

63. For any constant vector, show that the vector $\vec{a}$ represented by curl$(\vec{a}\times\vec{r})$ is always parallel to the vector $\vec{a},\vec{r}$ being the position vector of a point (x,y,z) measured from the origin. **[15 Marks]**

64. If $\vec{r}=x\hat{i}+y\hat{j}+x\hat{k}$ find the value(s) of in order that $r^n\vec{r}$ may be (i) solenoidal (ii) irrotational **[15 Marks]**

65. Determine $\int_c (ydx + zdy + xdz)$ by using Stoke's theorem, where C is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2, x + y = 2a$ that starts from the point $(2a,0,0)$ goes at first below the z-plane. **[15 Marks]**

2006

66. Find the values of constants a, b and c so that the directional derivative of the function $f = axy^2 + byz + cz^2x^2$ at the point $(1,2,-1)$ has maximum magnitude 64 in the direction parallel to z-axis. **[12 Marks]**

67. If $\overline{A} = 2\bar{i} + \overline{K}, \overline{B} = \bar{i} + \bar{j} + \overline{K}, \overline{C} = 4\bar{i} - 3\bar{j} - 7\overline{K}$ determine a vector $\overline{R}$ satisfying the vector equation $\overline{R} \times \overline{B} = \overline{C} \times \overline{B}$ & $\overline{R}.\overline{A} = 0$ **[15 Marks]**

68. Prove that $r^n \bar{r}$ is an irrotational vector for any value o f n but is solenoidal only if $n + 3 = 0$ **[15Marks]**

69. If the unit tangent vector $\bar{t}$ and binormal $\bar{b}$ make angles ϕ and ϕ respectively with a constant unit vector $\bar{a}$ prove that $\frac{\sin\theta}{\sin\phi}.\frac{d\theta}{d\phi}. = -\frac{k}{\tau}$. **[15 Marks]**

70. Verify Stokes' theorem for the function $\overline{F} = x^2\hat{i} - xy\hat{j}$ integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a$ and $y = a, a > 0$. **[15 Marks]**

2005

71. Show that the volume of the tetrahedron $ABCD$ is $\frac{1}{6}\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\overrightarrow{AD}$ Hence find the volume of the tetrahedron with vertices $(2,2,2),(2,0,0),(0,2,0)$ and $(0,0,2)$ **[12 Marks]**

72. Prove that the curl of a vector field is independent of the choice of coordinates **[12 Marks]**

73. The parametric equation of a circular helix is $r = a\cos u\hat{i} + a\sin u\hat{j} + cu\hat{k}$ where c is a constant and u is a parameter. Find the unit tangent vector $\hat{t}$ at the point u and the arc length measured form $u = 0$ Also find $\frac{d\hat{t}}{ds}$ where s is the arc length. **[15 Marks]**

74. Show that $\text{curl}\left(k \times \text{grad}\frac{1}{r}\right) + \text{grad}\left(k.\text{grad}\frac{1}{r}\right) = 0$ where r is the distance from the origin and k is the unit vector in the direction OZ **[15 Marks]**

75. Find the curvature and the torsion of the space curve **[15 Marks]**

76. Evaluate by Gauss divergence theorem, where S is the surface of the cylinder bounded by and **[15 Marks]**

2004

77. Show that if $\overline{A}$ and $\overline{B}$ are irrotational, then $\overline{A} \times \overline{B}$ is solenoidal. **[12 Marks]**

78. Show that the Frenet-Serret formulae can be written in the form $\frac{d\overline{T}}{ds} = \overline{\omega} \times \overline{T}, \frac{d\overline{N}}{ds} = \overline{\omega} \times \overline{N}$ & $\frac{d\overline{B}}{dx} = \overline{\omega} \times \overline{B}$, where $\overline{\omega} = \tau\overline{T} + k\overline{B}$. **[12 Marks]**

79. Prove the identity $\nabla\left(\overline{A}.\overline{B}\right) = \left(\overline{B}.\nabla\right)\overline{A} + \left(\overline{A}.\nabla\right)\overline{B} + \overline{B} \times \left(\nabla \times \overline{A}\right) + \overline{A} \times \left(\nabla \times \overline{B}\right)$ **[15 Marks]**

80. Derive the identity $\iiint_v \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right)dV = \iint_s \left(\phi\nabla\psi - \psi\nabla\phi\right)\hat{n}dS$ where V is the volume bounded by the closed surface S. **[15 Marks]**

81. Verify Stokes' theorem for $\hat{f} = (2x-y)\hat{i} - yz^2\hat{j}z\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. **[15 Marks]**

2003

82. Show that if a' b' and c' are the reciprocals of the non-coplanar vectors a, b and c , then any vector r may be expressed as $r = (r.a')a + (r.b')b + (r.c')c.$ **[12 Marks]**

83. Prove that the divergence of a vector field is invariant w, r, to co-ordinate transformations. **[12 Marks]**

84. Let the position vector of a particle moving on a plane curve be $r(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions. **[15 Marks]**

85. Prove the identity $\nabla A^2 = 2(A.\nabla)A + 2A\times(\nabla\times A)$ where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ **[15 Marks]**

86. Find the radii of curvature and torsion at a point of intersection of the surface $x^2 - y^2 = c^2, y = x\tanh\left(\frac{z}{c}\right).$ **[15 Marks]**

87. Evaluate $\int_s\int curtA.ds$ where S is the open surface $x^2+y^2-4x+4z=0, z\geq 0$ and $A = (y^2+z^2-x^2)\hat{i} + (2z^2+x^2-y^2)\hat{j} + (x^2+y^2-3z^2)\hat{k}.$ **[15 Marks]**

2002

88. Let $\overline{R}$ be the unit vector along the vector $\overline{r}(t)$ Show that $\overline{R}\times\frac{\overline{dR}}{dt} = \frac{\overline{r}}{r^2}\times\frac{\overline{dr}}{dt}$ where $r = \left|\overline{r}\right|$ **[12 Marks]**

89. Find the curvature k for the space curve $x = a\cos\theta, y = a\sin\theta, z = a\theta\tan\alpha$ **15 Marks]**

90. Show that $\left(cult\overline{v}\right) = grad\left(div\overline{v}\right) - \nabla^2\overline{v}$. **[15 Marks]**

91. Let D be a closed and bounded region having boundary S. Further, let f is a scalar function having second partial derivatives defined on it. Show that $\iint_s (\,fgradf\,).\hat{n}ds = \iiint_v \left[\left|gradf\right|^2 + f\nabla^2 f\right]dv$ Hence $\iint_s (\,fgradf\,).\hat{n}ds$ or otherwise evaluate for $f = 2x+y+2z$ over $s = x^2+y^2+z^2 = 4$ **[15 Marks]**

92. Find the values of constants a, b and c such that the maximum value of directional derivative of $f = axy^2 + byz + cx^2z^2$ at $(1,-1,1)$ is in the direction parallel to y-axis and has magnitude 6 **[15 Marks]**

2001

93. Find the length of the arc of the twisted curve $r = (3t, 3t^2, 2t^3)$ from the point $t=0$ to the point $t=1$ Find also the unit tangent t, unit normal n and the unit binormal b at $t=1$. **[12 Marks]**

94. Show that $curl\,\frac{a\times r}{r^3} = -\frac{a}{r^3} + \frac{3r}{r^5}(a.r)$ where a is constant vector. **[12 Marks]**

95. Find the directional derivative of $f = x^2yz^3$ along $x = e^{-t}, y = 1+2\sin t, z = t-\cos t$ at $t = 0$ **[15 Marks]**

96. Show that the vector field defined by $F = 2xyz^3 i + x^2z^3 j + 3x^2yz^2 k$ is irrotational. Find also the scalar u such that $F = grad\ u$ **[15 Marks]**

97. Verify Gauss' divergence theorem of $A = \left(4x, -2y^2, z^2\right)$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$. **[15 Marks]**

2000

98. In what direction from the point $(-1,1,1)$ is the directional derivative $f = x^2yz^3$ of a maximum? Compute its magnitude. **[12 Marks]**

99. Show that the covariant derivatives of the fundamental metric tensors $g_{ij}.g^{ij}, \delta^i{}_j$ Vanish
(ii) Show that simultaneity is relative in special relativity theory. **[6+6=12 Marks]**

100. Show that

$$(i)(A+B).(B+C)\times(C+A) = 2A.B\times C$$

$$(ii)\nabla\times(A\times B) = (B.\nabla)A - B(\nabla.A) - (A.\nabla)B + A(\nabla.B)$$

[7+8=15 Marks]

101. Evaluate $\iint_S F.Nds$ where $F = 2xyi + yz^2 j + xzk$ and S is the surface of the parallelepiped bounded by $x = 0, y = 0, z = 0.x = 2.y = 1$ and $z = 3$ **[15 Marks]**

102. If g_{ij} and γ_{ij} are two metric tensors defined at a point and $\left\{ {l \atop ij} \right\}$ and $\left[{l \atop ij} \right.$ are the corresponding Christoffel symbols of the second kind, then prove that $\left\{ {l \atop ij} \right\} - \left[{l \atop ij} \right.$ is a mixed tensor of the type $A^l{}_{ij}$ **[15 Marks]**

103. Establish the formula $E = mc^2$ the symbols have their usual meaning. **[15 Marks]**

1999

104. If $\overline{a}, \overline{b}\overline{c}$ are the position vectors of A, B, C prove that $\overline{a}\times\overline{b} + \overline{b} + \overline{c} + \overline{c}\times\overline{a}$ is vector perpendicular to the plane ABC **[20 Marks]**

105. If $\overline{f} = \nabla\left(x^3 + y^3 + z^3 - 3xyz\right)$ find $\nabla\times\overline{F}$. **[20 Marks]**

106. Evaluate $\int_c \left(e^{-x}\sin y dx + e^{-x}\cos y dy\right)$ (by Green's theorem), where C is the rectangle whose vertices are $(0,0), (\pi,0), \left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$ **[20 Marks]**

1998

107. If r_1 and r_2 are the vectors joining the fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ respectively to a variable point $P(x, y, z)$ then the values of grad$(r_1.r_2)$ and curl$(r_1\times r_2)$. **[20 Marks]**

108. Show that $(a\times b)\times c = a\times(a\times b)$ if either $b = 0$ (or any other vector is 0) or c is collinear with a or b is orthogonal to a and c (both) **[20 Marks]**

109. Prove that $\left\{ {i \atop ik} \right\} = \frac{\partial}{\partial x_k}\left(\log\sqrt{g}\right)$. **[20 Marks]**

1997

110. Prove that if $\vec{A}, \vec{B}$ and $\vec{C}$ are there given non-coplanar vectors $\vec{F}$ then any vector can be put in the form $F = \alpha\vec{B}\times\vec{C} + \beta\vec{C}\times\vec{A} + \gamma\vec{A}\times\vec{B}$ for given determine α, β, γ. **[20 Marks]**

111. Verify Gauss theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$ **[20 Marks]**

112. Prove that the decomposition of a tensor into a symmetric and an anti-symmetric part is unique. Further show that the contracted product $S_{ij}T_{ij}$ of a tensor T_{ij} with a symmetric tensor S_{ij} is independent of the anti-symmetric part of T_{ij}. **[20 Marks]**

1996

113. State and prove 'Quotient law' of tensors **[20 Marks]**

114. If $x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ show that

$(i)\vec{r} \times gradf(r) = 0$

$(ii)div(r^n\vec{r}) = (n+3)r^n$ **[20 Marks]**

115. Verify Gauss's divergence theorem for $\vec{F} = xyx\hat{i} + z^2\hat{j} + 2yz\hat{k}$ on the tetrahedron $x = y = z =, x + y + z = 1$. **[20 Marks]**

1995

116. Consider a physical entity that is specified by twenty-seven numbers A_{ijk} in given coordinate system. In the transition to anther coordinates system of this kind. Let $A_{ijk}B_{jk}$ transform as a vector for any choice of the anti-symmetric tensor. Prove that the quantities $A_{ijk} - A_{ijk}$ are the components of a tensor B_{jk} of third order. Is the component of tensor? Give reasons for your answer **[20 Marks]**

117. Let the reason V be bounded by the smooth surface S and let n denote outward drawn unit normal vector at a point on S. If ϕ is harmonic in V, show that $\int_s \frac{\partial\phi}{\partial n} ds = 0$ **[20 Marks]**

118. In the vector field $u(x)$ let there exists a surface $curl\upsilon$ on which $\upsilon = 0$. Show that, at an arbitrary point of this surface $curl\upsilon$ is tangential to the surface or vanishes. **[20 Marks]**

1994

119. Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n = -3$. **[20 Marks]**

120. If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$ evaluate $\iint_s \left(\Delta \times \vec{F}\right)\vec{n}ds$ Where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy plane. **[20 Marks]**

121. Prove that $\begin{Bmatrix} i \\ ik \end{Bmatrix} = \frac{\partial}{\partial x}\left(log\sqrt{g}\right)$. **[20 Marks]**

1993

122. Prove that the angular velocity or rotation at any point is equal to one half of the curl of the velocity vector V. **[20 Marks]**

123. Evaluate $\iint_S \Delta \times \vec{F}\vec{n}ds$ where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$ **[20 Marks]**

124. Show that $\dfrac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor or rank one **[20 Marks]**

1992

125. If $\vec{F}(x,y,z) = (y^2 + z^2)\vec{i} + (z^2 + x^2)\vec{j} + (x^2y^2)\vec{k}$ then calculate $\int_c \vec{f}\,\overrightarrow{dx}$ where C consist of

(i) The line segment form $(0,0,0)$ to $(1,1,1)$ (ii) the three line segments AB, BC and CD where A, B, C and D are respectively the points $(0,0,0),(1,0,0),(1,1,0)$ and $(1,1,1)$ (iii) the curve $\vec{x} + \overrightarrow{ui} + \overrightarrow{u^2 j} + \overrightarrow{u^2 k}, u$ from 0 to 1. **[20 Marks]**

126. If $\vec{a}$ and $\vec{b}$ are constant vectors, show that

$(i) div\left\{x \times (\vec{a} \times \vec{x})\right\} = -2\vec{x}\vec{a}$

$(ii) div\left\{x \times (\vec{a} \times \vec{x}) \times (\vec{b} \times \vec{x})\right\} = 2\vec{a}(\vec{b} \times \vec{x}) - 2\vec{b}(\vec{a} \times \vec{x})$ **[20 Marks]**

127. Obtain the formula $div\vec{A} = \dfrac{1}{\sqrt{g}}\dfrac{\partial}{\partial x^{-1}}\left\{\left(\dfrac{g}{g_{ij}}\right)^{1/2} A(i)\right\}$ where $A(i)$ are physical components of $\vec{A}$

and use it to derive expression of $div\vec{A}$ in cylindrical polar coordinates **[20 Marks]**