RAMANASRI IAS/IFoS Institute

Vector Analysis

IFoS (IFS) Previous Year Questions (PYQ) from 2020 to 2009

Ramanasri IFoS-IFS

IAS, UPSC, IFS, IFoS, CIVIL SERVICE MAINS EXAMS MATHS OPTIONAL STUDY MATERIALS

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Ramanasri IAS/IFoS(IFS) Maths Optional Vector Analysis PYQ 2020 to 2009

2020

1. Prove that for a vector $\vec{a}$, $\nabla(\vec{a}\ .\ \vec{r}) = \vec{a}$; where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}\,|$. Is there any restriction on $\vec{a}$? Further, show that $\vec{a}.\nabla\left(\vec{b}.\nabla\frac{1}{r}\right) = \frac{3.(\vec{a}.\vec{r})(\vec{b}.\vec{r})}{r^5} - \frac{\vec{a}.\vec{b}}{r^3}$. Give an example to verify the above. **[8 Marks]**

2. A tangent is drawn to a given curve at some point of contact. B is a point on the tangent at a distance 5 units from the point of contact. Show that the curvature of the locus of the point B is $\frac{\left[25k^2\tau^2(1+25k^2)+\left\{k+5\frac{dk}{ds}+25k^3\right\}\right]^{1/2}}{(1+25k^2)^{3/2}}$. Find the curvature and torsion of the curve $\vec{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$. **[15 Marks]**

3. Given a portion of a circular disc of radius 7 units and of height 1.5 units such that $x, y, z \geq 0$. Verify Gauss Divergence Theorem for the vector field $\vec{f} = (z, x, 3y^2 z)$ over the surface of the above-mentioned circular disc. **[15 Marks]**

4. Derive expression of ∇f in terms of spherical coordinates. Prove that $\nabla^2(fg) = f\nabla^2 g + 2\nabla f.\nabla g + g\nabla^2 f$ for any two vector point functions $f(r,\theta,\phi)$ and $g(r,\theta,\phi)$. Construct one example in three dimensions to verify this identity. **[10 Marks]**

2019

5. Let $\bar{r} = \bar{r}(s)$ represent a space curve. Find $\frac{d^3\bar{r}}{ds^3}$ in terms of $\bar{T}$, $\bar{N}$ and $\bar{B}$, where $\bar{T}$, $\bar{N}$ and $\bar{B}$ represent tangent, principal normal and binormal respectively. Compute $\frac{d\bar{r}}{ds}.\left(\frac{d^2\bar{r}}{ds^2} \times \frac{d^3\bar{r}}{ds^3}\right)$ in terms of radius of curvature and the torsion. **[8 Marks]**

6.

7. Verify stokes theorem for $\bar{V} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **[10 Marks]**

8. Derive $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in spherical coordinates and compute $\nabla^2 = \left(\frac{x}{(x^2+y^2+z^3)^{\frac{3}{2}}}\right)$ in spherical coordinates. **[15 Marks]**

9. Derive the Frenet-Serret formulae. Verify the same for the space curve $x = 3\cos t,\ y = 3\sin t,\ z = 4t.$ **[10 Marks]**

2018

10. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $f(r)$ is differentiable, show that $\text{div}\left[f(r)\vec{r}\right] = rf'(r) + 3f(r)$ Hence or otherwise show that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$ **[8 Marks]**

11. Let α be a unit-speed curve in R^3 with constant curvature and zero torsion. Show that α is (part of) a circle. **[10 Marks]**

12. For a curve lying on a sphere of radius a and such that the torsion is never 0, show that
$\left(\frac{1}{k}\right)^2+\left(\frac{k'}{k^2\tau}\right)^2=a^2$ **[10 Marks]**

13. Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force. Hence, find the scalar potential. Also find the work done in moving a particle of unit mass in the force field from $(1,-2,1)$ to $(3,1,4)$ **[15 Marks]**

2017

14. Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ and that $r^n \vec{r}$ is irrotational where $r=\left|\vec{r}\right|=\sqrt{x^2+y^2+z^2}$ **[8 Marks]**

15. Using stokes theorem, evaluate $\oint_C [(x+y)dx+(2x-z)dy+(y+z)dz]$ where C is the boundary of the triangle with vertices at $(2,0,0),(0,3,0)$ and $(0,0,6)$. **[15 Marks]**

16. Evaluate $\iint_S (\nabla\times\vec{f}).nds$ where S is the surface of the cone $z=2-\sqrt{x^2+y^2}$ above xy- plane and $\vec{f}=(x-z)\hat{i}+(x^2+yz)j-3xy^2)k,$ **[10 Marks]**

17. Find the curvature and torsion of the circular helix $\vec{r}=a(\cos\theta,\sin\theta,\theta\cot\beta)$ β is the constant angle at which it cuts its generators **[10 Marks]**

18. If the tangent to a curve makes a constant angle α with a fixed line, then prove that $k\cos\alpha\pm\tau\sin\alpha=0$ Conversely, if $\frac{k}{\tau}$ constant, then show that the tangent makes a constant angle with a fixed direction. **[10 Marks]**

2016

19. If E be the solid bounded by the xy plane and the paraboloid $z=4-x^2-y^2$, then evaluate $\iint_s \overline{F}.ds$ where S is the surface bounding the volume E and $\overline{F}=(zx\sin yz+x^3)\hat{i}+\cos yz\hat{j}+(3zy^2-e\lambda^2+y^2)\hat{k}$. **[8 Marks]**

20. Evaluate $\iint_s (\nabla\times\overline{f}).\hat{n}ds$ for $\overline{f}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ bounded by its projected on the xy plane. **[10 Marks]**

21. State stokes theorem. Verify the stokes' theorem for the function $\overline{f}=x\hat{i}+z\hat{j}+2y\hat{k}$, where C is the curve obtained by the intersection of the plane $z=x$ and the cylinder $x^2+y^2=1$ and S is the surface inside the intersected one. **[15 Marks]**

22. Prove that $\overline{a}\times(\overline{a}\times\overline{c})=(\overline{a}\times\overline{b})\times\overline{c}$, if and only if either $\overline{b}=\overline{0}$ or $\overline{c}$ is collinear with $\overline{a}$ or $\overline{b}$ is perpendicular to both $\overline{a}$ and $\overline{c}$. **[10 Marks]**

2015

23. Find the curvature and torsion of the curve $x = a\cos t, y = a\sin t, z = bt.$ **[8 Marks]**

24. Examine if the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + x^2yz^2\hat{k}$ is irrotational if so, find scalar potential ϕ such that $\vec{F} = \text{grad}\phi.$ **[10 Marks]**

25. Using divergence theorem, evaluate $\iint_s (x^3dydz + x^2ydzdx + x^2zdydx)$ where S is the surface of the sphere is $x^2 + y^2 + z^2 = 1.$ **[15 Marks]**

26. If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, evaluate $\iint_S \left(\nabla \times \vec{F}\right) \hat{n}\, dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. **[10 Marks]**

2014

27. For three vectors show that: $\bar{a} \times \left(\bar{b} \times \bar{c}\right) + \bar{b} \times \left(\bar{c} \times \bar{a}\right) + \bar{c} \times \left(\bar{a} \times \bar{b}\right) = 0.$ **[8 Marks]**

28. For the vector $A \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{x^2 + y^2 + z^2}$ examine if $\bar{A}$ is an irrotational vector. Then determine ϕ such that $\bar{A} = \nabla\phi$ **[10 Marks]**

29. Evaluate $\iint_S \nabla \times \overline{A}.\overline{n}ds$ for $\bar{A} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above xy plane **[15 Marks]**

30. Verify the divergence theorem for $\bar{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the region $x^2 + y^2 = 4, z = 0, z = 3$

[15 Marks]

2013

31. $\vec{F}$ Being a vector, prove that $\text{curl curl } \vec{F} =$ grad div $\vec{F} - \nabla^2\vec{F}$ where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} - + \dfrac{\partial^2}{\partial z^2}$

[8 Marks]

32. Evaluate $\int_s \vec{F}d\vec{s}\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$ **[13 Marks]**

33. Verify the Divergence theorem for the vector function $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c.$ **[14 Marks]**

2012

34. If $u = x+y+z, v = x^2+y^2+z^2, w = yz+zx+xy.$ prove that u , grad v and grad w are coplanar.

[8 Marks]

35. Find the value of $\iint_s \left(\vec{\nabla}\times\vec{F}\right).\vec{ds}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane $z=0$, when $\vec{F} = \left(y^2+z^2-x\right)\vec{i} + \left(z^2+x^2-y^2\right)\vec{j} + \left(x^2+y^2-z^2\right)\vec{k}.$ **[10 Marks]**

36. Find the value of the line integral over a circular path given by $x^2+y^2=a^2, z=0$, where the vector field, $\vec{F} = (\sin y)\vec{i} + x(1+\cos y)\vec{j}.$ **[10 Marks]**

2011

37. Evaluate the line integral $\oint_C (\sin x dx + y^2 dy - dz)$ where C is the circle **[10 Marks]**

38. Find the curvature torsion and the relation between the are length S and parameter u for the curve: $\vec{r} = \vec{r}(u) = 2\log\hat{i} + 4u\hat{j}(2u^2+1)\hat{k}$ **[10 Marks]**

39. Prove the vector identity curl $(\vec{f}\times\vec{g}) = \vec{f}\text{div }\vec{g} - \vec{g}\text{div }\vec{f} + (\vec{g}.\nabla) - (\vec{f}.\nabla)\vec{g}$ and verify it for the vectors $\vec{f} = x\hat{i} + z\hat{j} + y\hat{k}$ and $\vec{g} = y\hat{i} + z\hat{k}$ **[10 Marks]**

40. Verify Green's theorem in the plane for $\oint_C [3x^2-8y^2)dx + (4y-6xy)dy]$, where C is the boundary of the region enclosed by the curves $y=\sqrt{x}$ and $y=x^2$ **[10 Marks]**

41. The position vector $\vec{r}$ of a particle mass 2 units at any time t, referred to fixed origin and axes, is $\vec{r} = (t^2-2t)\hat{i} + \left(\frac{1}{2}t^2+1\right)\hat{j} + \frac{1}{2}t^2\hat{k}.$ At time $t=1$, find its kinetic energy, angular momentum, time rate of change of the angular moment and the moment of the resultant force, acting at the particle, about the origin. **[10 Marks]**

2010

42. Find the directional derivation of $\vec{V}^2$. where, $\vec{V} = xy^2\vec{i} + zy^2\vec{j} + xz^2\vec{k}$ at the point $(2,0,3)$ in the direction of the outward normal to the surface $x^2+y^2+z^2=14$ at the point $(3,2,1)$ **[8 Marks]**

43. (i) Show that $\vec{F} = (2xy+z^3)\vec{i} + x^2\vec{j} + 3z^2x\vec{k}$ is a conservative field. Find its scalar potential and also the work done in moving a particle from $(1,-2,1)$ to $(3,1,4)$ **[5 Marks]**

(ii) Show that, $\nabla^2 f(r) = \left(\frac{2}{r}\right)f'(r) + f''(r)$, where $r=\sqrt{x^2+y^2+z^2}$ **[5 Marks]**

44. Use divergence theorem to evaluate, $\iint_S (x^3 dydz + x^2 dzdx + x^2 zdydx)$, where S is the sphere, $x^2 + y^2 + z^2 = 1$. **[10 Marks]**

45. If $\vec{A} = 2y\vec{i} - z\vec{j} - x^2\vec{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4, z = 6,$ evaluate the surface integral, $\iint_S \vec{A}.\hat{n}\overrightarrow{ds}$ **[10 Marks]**

46. Use Green's theorem in a plane to evaluate the integral $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$, where C in the boundary of the surface in the xy- plane enclosed by, $y = 0$ and the semi-circle, $y = \sqrt{1 - x^2}$ **[10 Marks]**

2009

47. Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2 dy]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ **[10 Marks]**

48. Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational Find a scalar function ϕ such that $\vec{A} =$ grad ϕ. **[10 Marks]**

49. Let $\psi(x, y, z)$ be a scalar function. Find grad ψ and $\nabla^2\psi$ in spherical coordinates. **[8 Marks]**

50. Verify stokes' theorem for $\vec{A} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xzk,$ where S is the surface of the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the xy plane. **[12 Marks]**

51. Show that, If $\bar{r} = x(s)\hat{i} + y(s)\hat{j} + z(s)\hat{k}$ is a space curve, $\frac{d\bar{r}}{ds} \cdot \frac{d^2\bar{r}}{ds^2} \times \frac{d^3\bar{r}}{ds^3} = \frac{\tau}{\rho^2}$ where τ is the torsion and ρ the radius of curvature. **[10 Marks]**