RAMANASRI
IAS/IFoS Institute

Real Analysis

IFoS (IFS) Previous Year Questions (PYQ) from 2020 to 2009

Ramanasri Sir

IAS, UPSC, IFS, IFoS, CIVIL SERVICE MAINS EXAMS MATHS OPTIONAL STUDY MATERIALS

WEBSITE: MATHEMATICSOPTIONAL.COM
CONTACT: 8750706262 | 8750706363

2020

1. (i) If $u = u(y-z, z-x, x-y)$ then find the value of $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$. (ii) If $u(x,y,z)=\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}$, then find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$. **[8 Marks]**

2. Evaluate the integral $\iint_R (x-y)^2\cos^2(x+y)dxdy$, where R is the rhombus with successive vertices at $(\pi,0),(2\pi,\pi),(\pi,2\pi)$ and $(0,\pi)$. **[8 Marks]**

3. Show that the sequence of functions $\{f_n(x)\}$, where $f_n(x)=nx(1-x)^n$, does not converge uniformly on $[0,1]$. **[15 Marks]**

4. Find the extreme values of $f(x,y,z)=2x+3y+z$ such that $x^2+y^2=5$ and $x+z=1$. **[10 Marks]**

2019

5. Show that the function $\mathrm{f}(\mathrm{x})=\sin\left(\frac{1}{\mathrm{x}}\right)$ is continuous and bounded in $(0,\ 2\pi)$, but it is not uniformly continuous in $(0,\ 2\pi)$ **[8 Marks]**

6. Test the Riemann integrability of the function f defined by $\mathrm{f}(\mathrm{x})=\begin{cases}0 & \text{when x is rational}\\ 1 & \text{when x is irrational}\end{cases}$ on the interval $[0,\ 1]$. **[8 Marks]**

7. Show that the integral $\int_0^{\pi/2}\log\sin\mathrm{x}\,\mathrm{dx}$ is convergent and hence evaluate it. **[15 Marks]**

8. Show that the sequence $\{\tan^{-1}\mathrm{nx}\}, \mathrm{x}\ge 0$ is uniformly convergent on any interval $[\mathrm{a},\ \mathrm{b}], \mathrm{a}>0$ but is only point wise convergent on $[0,\ \mathrm{b}]$. **[15 Marks]**

2018

9. Consider the function f defined by $f(x,y)=\begin{cases}xy\frac{x^2-y^2}{x^2+y^2}, & \text{where } x^2+y^2\neq 0\\ 0 & \text{, where } x^2+y^2=0\end{cases}$ Show that $f_{xy}\neq f_{yx} at (0,0)$ **[10 Marks]**

10. Prove that $\int_0^\infty \cos x^2 dx=\int_0^\infty \sin x^2 dx=\frac{1}{2}\sqrt{\frac{\pi}{2}}$ **[10 Marks]**

11. Find the minimum value of $x^2+y^2+z^2$ subject to the condition $ax+by+cz=p$ **[10 Marks]**

12. Show that the improper integral $\int_0^\infty \frac{\sin\frac{1}{x}}{\sqrt{x}}dx$ is convergent. **[10 Marks]**

13. Show that $\iint_R x^{m-1}y^{n-1}(1-x-y)^{l-1}dx\,dy = \dfrac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n)}$; $l, m, n > 0$ taken over R : the triangle bounded by $x=0, y=0, x+y=1$ **[10 Marks]**

14. Let $f_n(x) = \dfrac{x}{n+x^2}, x \in [0,1]$ Show that the sequence $\{f_n\}$ is uniformly convergent on $[0,1]$ **[8 Marks]**

2017

15. Evaluate $\int_{x=0}^{\infty}\int_{y=0}^{x} xe^{-x^2/y}dy\,dx$ **[8 Marks]**

16. Find the volume of the region common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$. **[8 Marks]**

17. Evaluate $f_{xy}(0,0)$ and $f_{yx}(0,0)$ given that $f(x)=\begin{cases} x^2\tan^{-1}\dfrac{y}{x} - y^2\tan^{-1}\dfrac{x}{y} & \text{if } xy \neq 0 \\ 0 & \text{,otherwise} \end{cases}$ **[10 Marks]**

18. Find the maximum and minimum values of $x^2+y^2+z^2$ subject to the condition $\dfrac{x^2}{4}+\dfrac{y^2}{5}+\dfrac{z^2}{25}=1$. **[10 Marks]**

19. Prove that $\int_0^{\infty}\dfrac{\sin x}{x}dx$ is convergent but not absolutely convergent. **[12 Marks]**

20. A function $f:\mathbb{R}\to\mathbb{R}$ is defined as below: $f(x)=\begin{cases} x \text{ if } x \text{ is rational} \\ 1\text{-}x \text{ if } x \text{ isirrational} \end{cases}$ prove that f is continuous at $x=\dfrac{1}{2}$ but discontinuous at all other points in $\mathbb{R}$. **[10 Marks]**

2016

18. Examine the Uniform convergence of $f_n(x) = \dfrac{\sin(nx+n)}{n}, \forall x \in \mathbb{R}, n = 1,2,3,...$ **[8Marks]**

19. Find the maxima and minima of the function $f(x,y) = x^3+y^3-3x-12y+20$ **[8 Marks]**

20. If $f_n(x) = \dfrac{3}{x+n}, 0 \le x \le 2$, state with reasons whether $\{f_n\}_n$ converges uniformly on $[0,2]$ or not **[10 Marks]**

21. Examine the continuity of $f(x,y)=\begin{cases} \dfrac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)}, & (x,y)\neq(0,0) \\ \dfrac{1}{2} & ,(x,y)=(0,0) \end{cases}$ at the point $(0,0)$ **[8 Marks]**

22. If $u(x,y) = \cos^{-1}\left\{\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right\}, 0<x<1, 0<y<1$ then find the value of $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$ **[10 Marks]**

23. Evaluate the integral $\int_0^2\int_0^{\frac{y^2}{2}} \dfrac{y}{(x^2+y^2+1)^{\frac{1}{2}}}dxdy.$ **[12marks]**

24. Evaluate the integral $\int_0^\infty \frac{dx}{\sqrt{x}(1+x)}$ **[8 Marks]**

2015

25. Let $\sum_{n=1}^{\infty} a_n$ be an absolutely convergent series of real numbers. Suppose $\sum_{n=1}^{\infty} a_{2n} = \frac{9}{8}$ and $\sum_{n=0}^{\infty} a_{2n+1} = \frac{-3}{8}$. What is $\sum_{n=1}^{\infty} a_n$? Justify your answer (Majority of marks is for the correct justification) **[8 Marks]**

26. Let $X = (a,b]$. Construct a continuous function $f : X \to \mathbb{R}$ (set of real numbers) which is unbounded and not uniformly continuous on X. Would your function be uniformly continuous on $[a+\varepsilon, b], a+\varepsilon < b$? why?

27. Let $f_n(x) = \frac{x}{1+nx^2}$ for all real x. Show that f_n converges uniformly to a function f. What is f ? Show that $x \neq 0, f'_n(x) \to f'(x)$ but $f'_n(0)$ does not converge to $f'(0)$. Show that the maximum value $|f_n(x)|$ can take is $\frac{1}{2\sqrt{n}}$ **[13 Marks]**

28. Compute the double integral which will give the area of the region between the y- axis, the circle $(x-2)^2 + (y-4)^2 = z^2$ and the parabola $2y = x^2$. Compute the integral and find the area. **[15 Marks]**

2014

29. Let f be defined on $[0,1]$ as $f(x) = \begin{cases} \sqrt{1-x^2}, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$ Find the upper and lower Riemann integrals of f over $[0,1]$. **[8 Marks]**

30. Show that the functions $f(x) = \sin\frac{1}{x}$ continuous but not uniformly continuous on $(0,\pi)$ **[15 Marks]**

31. Change the order of integration and evaluate $\int_{-2}^{1}\int_{y^2}^{2-y} dxdy$ **[15 Marks]**

32. Show that the function $f(x) = \sin x$ Riemann integrable in any interval $[0,t]$ by taking the partition $P = \left\{0, \frac{t}{n}, \frac{2t}{n}, \frac{3t}{n}, \ldots, \frac{nt}{n}\right\}$ and $\int_0^t \sin x dx = 1 - \cos t$. **[10 Marks]**

2013

33. Evaluate: $\lim_{x\to 0}\left(\frac{e^{ax} - e^{bx} + \tan x}{x}\right)$ **[10 Marks]**

34. Show that the function $f(x) = x^2$ uniformly continuous in $(0,1)$ but not in $\mathbb{R}$ **[13 Marks]**

35. Find the of the region between the x-axis and $y=(x-1)^3$ from $x=0$ to $x=2$ **[13 Marks]**

2012

36. Show that the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\begin{cases}1, & x \text{ is irrational}\\ -1, & x \text{ is ratinal}\end{cases}$ is discontinuous at every point in $\mathbb{R}$ **[10 Marks]**

37. Show that the function:

$$u=x^2+y^2+z^2$$
$$v=x+y+z$$
$$w=yz+zx+xy$$

Are not independent of one another **[10 Marks]**

38. If $u=x^2\tan^{-1}\left(\frac{x}{y}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2u$. **[13 Marks]**

39. Find the volume of the solid bounded above by the parabolic cylinder $z=4-y^2$ and bounded below by the elliptic paraboloid $z=x^2+3y^2$ **[13 Marks]**

40. Examine the series $\sum_{n=1}^{\infty}u_n(x)=\sum_{n=1}^{\infty}\left[\frac{nx}{1+n^2x^2}-\frac{(n-1)x}{1+(n-1)^2x^2}\right]$ for uniform convergence. Also, with the help of this example, show that the condition of uniform convergence $\sum_{n=1}^{\infty}u_n(x)$ of is sufficient but not necessary for the sum $S(x)$ of the series to be continuous **[13 Marks]**

2011

41. Determine whether $f(x)=2x\sin\frac{1}{x}-\cos\frac{1}{x}$ is Riemann- integrable on $[0,1]$ and justify your answer

[10 Marks]

42. Let the function f be defined by $f(x)=\frac{1}{2^t}$, when $\frac{1}{2^{t+1}}<x\le\frac{t}{2^t}(t=0,1,2,3,\ldots)f(0)=0$ is f integrable, on $[0,1]$? if f is integrable, then evaluate $\int_0^1 f dx$. **[13 Marks]**

43. Sketch the image of the infinite strip $1<y<2$ under the transformation $w=\frac{1}{z}$ **[14 Marks]**

44. Examine the convergence of $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ and evaluate, if possible. **[13 Marks]**

45. Evaluate $\iint\sqrt{4x^2-y^2}\,dxdy$ over the triangle formed by the straight lines $y=0, x=1, y=x$.

[13 Marks]

2010

46. A captain of a cricket team has to allot four middle-order batting position to four batsmen. The average numbers of runs scored by each batsman these positions are as follows. Assign each batsman his batting position for maximum performance:

Batsman \ Batting Position	IV	V	VI	VII
A	40	25	20	35
B	36	30	24	40
C	38	30	18	40
D	40	23	15	33

[10 Marks]

47. If $f: R \to R$ is such that $f(x+y) = f(x)f(y)$ for all x, y in R and $f(x) \neq 0$ for any x in R, Show that $f'(x) = f(x)$ for all x in R given that $f'(0) = f(0)$ and the function is differentiable for all x in R

[10 Marks]

48. A rectangular box open at the top is to have a surface area of 12 square units, Find the dimension of the box so that the volume is maximum. **[13 Marks]**

49. Evaluate $\iint_R (x-y+1)\, dx\, dy$ where R is the region inside the unit square in which $x + y \geq \frac{1}{2}$

[13 Marks]

2009

50. Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $[0,1]$ **[10 Marks]**

51. Find the dimension of the largest rectangular parallelepiped that has three faces in the coordinate planes and one vertex in the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ **[14 Marks]**

52. Evaluate $\iint xy(x+y)dx\, dy$ over the area between $y = x^2$ and $y = x$. **[13 Marks]**