

Partial Differential Equations (PDE)

IFoS (IFS) Previous Year
Questions (PYQ) from
2020 to 2009

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IAS, UPSC, IFS, IFoS, CIVIL
SERVICE MAINS EXAMS
MATHS OPTIONAL STUDY
MATERIALS

Ramanasri IAS/IFoS(IFS) Maths Optional Partial Differential Equations (PDE) PYQ 2020 to 2009

2020

1. Construct a partial differential equation of all surfaces of revolution having the z -axis as the axis of rotation. [8 Marks]
2. Find the general solution of the partial differential equation $p \tan x + q \tan y = \tan z$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. [8 Marks]
3. Find the general solution and singular solution of the partial differential equation $6yz - 6pxy - 3qy^2 + pq = 0$ [10+5=15 Marks]
4. Find the solution of the following differential equation: $2 \frac{\partial^2 z}{\partial x^2} + 5 \frac{\partial^2 z}{\partial x \partial y} + 3 \frac{\partial^2 z}{\partial y^2} = ye^x$ [10 Marks]

2019

5. Find the solution of the equation: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$ [8 Marks]
6. Find a complete integral of the equation by Charpit's method $p^2 x + q^2 y = z$. Here $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. [8 Marks]
7. Test the integrability of the equation $z(z + y^2) dx + z(z + x^2) dy - xy(x + y) dz = 0$ If integrable, then find its solution. [15 Marks]
8. Find the equation of the system of curves on the cylinder $2y = x^2$ orthogonal to its intersections with the hyperboloids of the one-parameter system $xy = z + c$. [15 Marks]

2018

9. Find the complete integral of the partial differential equation $(p^2 + q^2)x = zp$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$ here $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ [12 Marks]
10. Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ if the solution of the above equation represents a sphere, what will be the coordinates of its centre? [8 Marks]
11. Find a real function V of x and y satisfying $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$ and reducing to zero, when $y = 0$ [10 Marks]
12. Find the partial differential equation of all planes which are at a constant distance a from the origin [10 Marks]

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2017

13. Form the partial differential equation by eliminating arbitrary function ϕ and ψ from the relation $z = \phi(x^2 - y) + \psi(x^2 + y)$ [8 Marks]
14. Solve the partial differential equation $(x - y)\frac{\partial z}{\partial x} + (x + y)\frac{\partial z}{\partial y} = 2xz$. [8 Marks]
15. Find the surface which is orthogonal to the family of surface $z(x + y) = c(3z + 1)$ and which passes through the circle $x^2 + y^2 = 1, z = 1$, [8 Marks]
16. Find complete integral $xp - yq = xqf(z - px - qy)$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [12 Marks]
17. Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$. [12 Marks]
18. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$ It is released from rest from this position, find the displacement $y(x, t)$ [12 Marks]

2016

19. Obtain the partial differential equation governing the equations $\phi(u, v) = 0, u = xyz, v = x + y + z$ [8 Marks]
20. Find the general solution of the partial differential equation $xy^2 \frac{\partial z}{\partial x} + y^3 \frac{\partial z}{\partial y} = (zxy^2 - 4x^3)$ [8 Marks]
21. Find the general solution of the partial differential equation $xy^2 p + y^3 q = (zxy^2 - 4x^3)$ $\left[p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y} \right]$ [10 Marks]
22. Find the particular integral of $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2x \cos y$ [10 Marks]

2015

23. Find the solution of the equation $u_{xx} - 3u_{xy} + u_{yy} = \sin(x - 2y)$ [10 Marks]
24. Solve the differential equation $u_x^2 - u_y^2$ by variable separation method. [12 Marks]
25. Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$ subject to the conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = \sin \pi x, 0 < x < 1$. [14 Marks]

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2014

26. Show that the general solution of the PDE $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 z}{\partial t^2}$ is of the form $Z(x, y) = F(x + ct) + G(x - ct)$, where F and G are arbitrary function. [8 Marks]
27. Verify that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and find its primitive. [10 Marks]
28. Solve $(D - 3D' - 2)^2 z = 2e^{2x} \cot(y + 3x)$ [15 Marks]

2013

29. Eliminate the arbitrary function from the given equation $f(x^2 + y^2 + z^2, x + y + z) = 0$ [12 Marks]
30. Solve the PDE: $xu_x + yu_y + zu_z = xyz$ [12 Marks]
31. Rewrite the hyperbolic equation $x^2 u_{xx} - y^2 u_{yy} = 0$ ($x > 0, y > 0$) in canonical form. [16 Marks]
32. Find the solution of the equation $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$ that passes through the circle $x^2 + y^2 = 1, u = 1$ [16 Marks]
33. Solve the following heat equation, using the method of separation of variable:
 $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$ Subject to the condition $u = 0$ At $x = 0$ and $x = 1$, for $t > 0$
 $u = 4x(1 - x)$, At $t = 0$ for $0 \leq x \leq 1$ [16 Marks]

2012

34. Solve $(D^3 D'^2 + D^2 D'^3)z = 0$, where D stand for $\frac{\partial}{\partial x}$ and D' stands for $\frac{\partial}{\partial y}$. [10 Marks]
35. Using Method of separation of Variable, Solve Laplace equation in three dimensions. [13 Marks]
36. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ using Lagrange's method. [13 Marks]

2011

37. Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to its canonical form and solve. [10 Marks]
38. Find the complementary function and particular integral of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$ [12 Marks]

2010

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39. Find the general solution of $x(y^2 + z)p + y(x^2 + z)q = z(x^2 - y^2)$ [10 Marks]
40. Solve $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ given the conditions (i) $u(0, t) = u(\pi, t) = 0, t > 0$ (ii) $u(x, 0) = \sin 2x, 0 < x < \pi$ [16 Marks]
41. Find the general solution of $(D - D' - 1)(D - D' - 2)z = e^{2x-y} + \sin(3x + 2y)$ [13 Marks]
42. Solve that initial value problem $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$ for $x = 0.1$ Euler's method. [13 Marks]

2009

43. Find complete and singular integrals of $(p^2 + q^2)y = qz$ [10 Marks]
44. A rod of length l with insulated sides is initially at a uniform temperature u_0 its ends are suddenly cooled to $0^\circ C$ and are kept at that temperature. Find the temperature in the rod at any time t [14 Marks]
45. Find the general solution of $\{D^2 - DD' - 2D'^2 + 2D + 2D'\}z = e^{2x+3y} + xy + \sin(2x + y)$ [13 Marks]