

Ordinary Differential  
Equations (ODE)

IFoS (IFS) Previous Year  
Questions (PYQ) from  
2020 to 2009

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IAS, UPSC, IFS, IFoS, CIVIL  
SERVICE MAINS EXAMS MATHS  
OPTIONAL STUDY MATERIALS

# Ramanasri IAS/IFoS(IFS) Maths Optional

## Ordinary Differential Equations PYQ 2020 to 2009

### 2020

1. Solve the initial value problem:  $(2x^2 + y)dx + (x^2y - x)dy = 0$ ,  $y(1) = 2$ . [8 Marks]
2. Solve the differential equation  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 16x - 12e^{2x}$  [8 Marks]
3. Find one solution of the differential equation  $(x^2 + 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$  by inspection and using that solution determine the other linearly independent solution of the given equation. Obtain the general solution of the given differential equation. [10 Marks]
4. Solve the differential equation  $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x^2)}$ . [15 Marks]
5. Reduce the differential equation  $xp^2 - 2yp + x + 2y = 0$ ,  $\left(p = \frac{dy}{dx}\right)$ , to Clairaut's form and obtain its complete primitive. Also, determine a singular solution of the given differential equation [15 Marks]

### 2019

6. Solve the differential equation  $(px - y)(py + x) = h^2p$ , where  $p = y'$ . [8 Marks]
7. Solve the differential equation  $(D^2 + 1)y = x^2 \sin 2x$ ;  $D \equiv \frac{d}{dx}$ . [8 Marks]
8. Solve by the method of variation of parameters the differential equation  $x''(t) - \frac{2x(t)}{t^2} = t$ , Where  $0 < t < \infty$  [15 Marks]
9. Find the general solution of the differential equation  $\ddot{x} + 4x = \sin^2 2t$  Hence find the particular solution satisfying the conditions  $x\left(\frac{\pi}{8}\right) = 0$  and  $\dot{x}\left(\frac{\pi}{8}\right) = 0$  [15 Marks]
10. Find the general solution of the differential equation  $(x - 2)y'' - (4x - 7)y' + (4x - 6)y = 0$  [10 Marks]

### 2018

11. Find the complementary function and particular integral for the equation  $\frac{d^2y}{dx^2} - y = xe^x + \cos^2 x$  and hence the general solution of the equation [8 Marks]
12. Solve  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \log x (x \geq 0)$  by the method of variation of parameters [8 Marks]
13. Solve the differential equation  $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$  [10 Marks]
14. Solve:  $\frac{dy}{dx} = \frac{4x + 6y + 5}{3y + 2x + 4}$  [10 Marks]

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## Ordinary Differential Equations PYQ 2020 to 2009

### 2017

15. Solve  $(2D^2 - 7D + 7D - 2)y = e^{-8x}$  where  $D = \frac{d}{dx}$  [8 Marks]
16. Solve the differential equation  $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$  [8 Marks]
17. Solve the differential equation  $\left(\frac{dy}{dx}\right)^2 + 2 \cdot \frac{dy}{dx} \cdot y \cot x = y^2$  [15 Marks]
18. Solve the differential equation  $e^{3x} \left(\frac{dy}{dx} - 1\right) + \left(\frac{dy}{dx}\right)^3 e^{2y} = 0$ . [10 Marks]
19. Solve  $\frac{d^2y}{dx^2} + 4y = \tan 2x$  by using the method of variation of parameter. [10 Marks]

### 2016

20. Obtain the curve which passes through (1, 2) and has a slope =  $\frac{-2xy}{x^2 + 1}$ , obtain one asymptote to the curve. [8 Marks]
21. Solve the DE to get the particular integral of  $\frac{d^4y}{dx^4} + 2 \frac{d^2y}{dx^2} + y = x^2 \cos x$  [8 Marks]
22. Using the method of variation of parameters solve  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x$  [10 Marks]
23. Obtain the singular solution of the differential equation  $y^2 - 2pxy + p^2(x^2 - 1) = m^2, p = \frac{dy}{dx}$
24. Solve the differential equation  $\frac{dy}{dx} - y = y^2(\sin x + \cos x)$  [10 Marks]

### 2015

25. If  $\sqrt{x+y} + \sqrt{y-x} = c$ , Find  $\frac{d^2y}{dx^2}$  [10 Marks]
26. Reduce the differential equation  $x^2 p^2 + yp(2x+y) + y^2 = 0, p = \frac{dy}{dx}$  Clairaut's form. Hence find the singular solution of the equation. [8 Marks]
27. Solve the differential equation  $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$  [8 Marks]

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## Ordinary Differential Equations PYQ 2020 to 2009

28. Solve  $x \frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3 \sin x^2$  by changing the independent variable. [10 Marks]

29. Solve  $(D^4 + D^2 + 1)y = e^{-x/2} \cos\left(\frac{x\sqrt{3}}{2}\right)$ , where  $D = \frac{d}{dx}$ . [10 Marks]

### 2014

30. Solve the differential equation:  $y = 2px + p^2y$ ,  $p = \frac{dy}{dx}$  and obtain the non-singular solution. [8 Marks]

31. Solve:  $\frac{d^4y}{dx^4} - 16y = x^4 + \sin x$ . [8 Marks]

32. Solve the following differential equation:  $\frac{dy}{dx} = \frac{2y}{x} + \frac{x^3}{y} + x \tan \frac{y}{x^2}$  [10 Marks]

33. Solve the DE  $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$  [10 Marks]

34. Solve by the method of variation of parameters:  $y'' + 3y' + 2y = x + \cos x$  [10 Marks]

### 2013

35. Solve  $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$  [8 Marks]

36. Solve the differential equation  $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$  by changing the dependent variable. [13 Marks]

37. Solve  $(D^3 + 1)y = e^{x/2} \sin\left(\frac{\sqrt{3}}{2}x\right)$  where  $D = \frac{d}{dx}$  [13 Marks]

38. Apply the method of variation of parameters to solve  $\frac{d^2y}{dx^2} - y = 2(1 + e^x)^{-1}$  [13 Marks]

### 2012

39. Solve  $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$  [8 Marks]

40. Solve and find the singular solution of  $x^3p^2 + x^2py + a^3 = 0$  [8 Marks]

41. Solve  $x^2y \frac{d^2y}{dx^2} + \left(x \frac{dy}{dx} - y\right)^2 = 0$ . [10 Marks]

# Ramanasri IAS/IFoS(IFS) Maths Optional

## Ordinary Differential Equations PYQ 2020 to 2009

42. Solve  $\frac{d^4 y}{dx^4} + 2\frac{d^2 y}{dx^2} + y = x^2 \cos x$  [10 Marks]
43. Solve  $x = y \frac{dy}{dx} - \left(\frac{dy}{dx}\right)^2$ . [10 Marks]
44. Solve  $x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = (1-x)^{-2}$ . [10 Marks]

### 2011

45. Find the family of curve whose tangents form an angle  $\frac{\pi}{4}$  with hyperbolas  $xy = c$ . [10 Marks]
46. Solve  $\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = \sec x \cdot e^x$  [10 Marks]
47. Solve  $p^2 + 2p y \cot x = y^2$ , where  $p = \frac{dy}{dx}$  [10 Marks]
48. Solve  $(x^4 D^4 + 6x^3 D^3 + 9x^2 D^2 + 3x D + 1)y = (1 + \log x)^2$ , where  $D \equiv \frac{d}{dx}$  [15 Marks]
49. Solve  $(D^4 + D^2 + 1)y = ax^2 + be^{-x} \sin 2x$ , where  $D \equiv \frac{d}{dx}$  [15 Marks]

### 2010

50. Show that  $\cos(x+y)$  is an integration factor of  $y dx + [y + \tan(x+y)] dy = 0$ . Hence Solve it [8 Marks]
51. Solve  $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$  [8 Marks]
52. Solve the following differential equation  $\frac{dy}{dx} = \sin^2(x-y+6)$  [8 Marks]
53. Find the general solution of  $\frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + (x^2+1)y = 0$  [12 Marks]
54. Solve  $\left(\frac{d}{dx} - 1\right)^2 \left(\frac{d^2}{dx^2} + 1\right)^2 y = x + e^x$  [10 Marks]
55. Solve by the matrix of variation of parameters the following equation  
 $(x^2 - 1) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = (x^2 - 1)^2$  [10 Marks]

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## Ordinary Differential Equations PYQ 2020 to 2009

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2009

56. Solve:  $\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$  [10 Marks]
57. Find the 2<sup>nd</sup> order ODE for which  $e^x$  and  $x^2 e^x$  are solutions. [10 Marks]
58. Solve:  $(y^3 - 2yx^2)dx + (2xy^2 - x^3)dy = 0$  [10 Marks]
59. Solve:  $\left(\frac{dy}{dx}\right)^2 - 2\frac{dy}{dx} \cosh x + 1 = 0$  [8 Marks]
60. Solve:  $\frac{d^3 y}{dx^3} + 3\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + y = x^2 e^{-x}$  [10 Marks]
61. Show that  $e^{x^2}$  is a solution of  $\frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 2)y = 0$  Find a second independent solution. [12 Marks]