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Modern Algebra

IFoS (IFS) Previous Year Questions (PYQ) from 2020 to 2009

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IAS, UPSC, IFS, IFoS, CIVIL SERVICE MAINS EXAMS MATHS OPTIONAL STUDY MATERIALS

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2020

1. Let p be prime number. Then show that $(p-1)!+1 \equiv 0(\text{mod } p)$. Also, find the remainder when $6^{44}.(22)!+3$ is divided by 23. **[8 Marks]**

2. Let R be a non –zero commutative ring with unity. Show that M is a maximal ideal in a ring R if and only if $\frac{R}{M}$ is a field. **[10 Marks]**

3. Let G be a finite group and let p be a prime. If p^m divides order of G, then show that G has a subgroup of order p^m, where m is a positive integer. **[15 Marks]**

4. Let K be a finite filed. Show that the number of elements in K is p^n, where p is a prime, which is characteristic of K and $n \geq 1$ is an integer. Also, prove that $\frac{\mathbb{Z}_3[x]}{(X^2+1)}$ is a field. How many elements does this field have? **[15 Marks]**

2019

5. Let R be an integral domain. Then prove that Ch R (characteristic of R) is 0 or a prime. **[8 Marks]**

6.

7. Let I and J be ideals in a ring R. Then prove that the quotient ring $(\mathrm{I}+\mathrm{J})/\mathrm{J}$ is isomorphic to the quotient ring $\mathrm{I}/(\mathrm{I} \cap \mathrm{J})$. **[10 Marks]**

8. If in the group G, $a^5 = e$, $aba^{-1} = b^2$ for some $a, b \in \mathrm{G}$, find the order of b. **[10 Marks]**

9. Show that the smallest subgroup V of A_4 containing (1, 2) (3, 4), (1, 3) (2, 4) and (1, 4) (2, 3) is isomorphic to the Klein 4-group. **[10 Marks]**

2018

10. Prove that a non-commutative group of order $2n$, where n is an odd prime, must have a subgroup of order n. **[8 Marks]**

11. Find all the homomorphisms from the group $(\mathbb{Z},+)$ to $(\mathbb{Z}_4,+)$ **[10 Marks]**

12. Let R be a commutative ring with unity. Prove that an ideal P of R is prime if and only if the quotient ring R/P is an integral domain. **[10 Marks]**

13. Show by an example that in a finite commutative ring, every maximal ideal need not be prime. **[10 Marks]**

14. Let H be a cyclic subgroup of a group G. If H be a normal subgroup of G, prove that every subgroup of H is a normal subgroup of G **[10 Marks]**

2017

15. Prove that every group of order four is Abelian. **[8 Marks]**

16. Let G be the set of all real numbers except -1 and define $a*b=a+b+ab\forall a,b\in G$ Examine if G is an Abelian group under $*$ **[10 Marks]**

17. Let H and K are two finite normal subgroups of co-prime order of a group G. Prove that $hk=kh\ \forall h\in H$ and $k\in K$. **[10 Marks]**

18. Let A be an ideal of a commutative ring R and $B=\left\{x\in R: x^n\in A \text{ for some positive interger n}\right\}$ Is B an ideal of R? Justify your answer. **[10 Marks]**

19. Prove that the ring $\mathbb{Z}[i]=\left\{a+ib: a,b\in\mathbb{Z}, i=\sqrt{-1}\right\}$ of Gaussian integers is a Euclidean domain **[10 Marks]**

2016

20. Prove that the set of all bijective functions form a non- empty set X onto itself is a group with respect to usual composition of functions. **[8 Marks]**

21. Show that in the ring $R=\{a+b\sqrt{-5}\,|a,b$ are integers}, the elements $\alpha=3$ and $\beta=1+2\sqrt{-5}$ are relatively prime but $\alpha\gamma$ and $\beta\gamma$ have no $g.c.d$ in R, where $\gamma=7(1+2\sqrt{5})$ **[10 Marks]**

22. Let G be a group of order pq, where p and q are prime numbers such that $p>q$ and $q\chi(p-1)$ Then prove that G is cyclic. **[15marks]**

23. Show that any non-abelian group of order 6 is isomorphic to the symmetric group S_3 **[15marks]**

2015

24. If in a group G there is an element a of order 360, what is the order of a^{220} ? Show that if G is cyclic group of order n and m divides n, then G has a subgroup of order m **[10 Marks]**

25. If p is a prime number and e a positive integer, what are the elements 'a' in the ring $\mathbb{Z}_pe$ of integer modulo p^e such that $a^2=a$? Hence(or otherwise) determine the elements in $\mathbb{Z}_{35}$ such that $a^2=a$ **[14 Marks]**

26. What is the maximum possible order of a permutation in S_8, the group of permutations on the eight numbers $\{1,2,3,...,8\}$? Justify your answer (Majority of marks will be given for the justification). **[13 Marks]**

2014

27. If G is a group which $(a\cdot b)=a^4\cdot b^4, (a\cdot b)^5=a^5\cdot b^5$ and $(a.b)^6=a^6.b^6$, for all $a,b\in G$, then prove that G is Abelian. **[8 Marks]**

28. Let j_n be the set of integers mod n then prove then j_n is a ring under the operation of addition and multiplication mod n under what continuous on n, j_n is a field? Justify you answer. **[10 Marks]**

29. Let R be an integral domain with unity. Prove that the units of R and $R|x|$ are same. **[10 Marks]**

2013

30. Prove that if every element of group $(G, 0)$ be its own inverse, then it is an abelian group **[10 Marks]**

31. Show that any integral domain is a field **[13 Marks]**

32. Every field is an integral domain –Prove it **[13 Marks]**

33. Prove that (i)The intersection of two ideals is an ideal (ii) A field has proper ideals **[14 Marks]**

2012

34. Show that every field is without zero divisor. **[10 Marks]**

35. Show that in a symmetric group S_3, there are four elements σ satisfying σ^2 = Identity. And three elements satisfying σ^3 = Identity. **[13 Marks]**

36. If R is an integral domain, show that the polynomial ring $R[x]$ also an integral domain. **[14 Marks]**

2011

37. Let G be a group, and x and y be any two elements of G .If $y^5 = e$ and $yxy^{-1} = x^2$, then Show that $O(x) = 31,$ where e is the identity element of G and $x \neq e.$ **[10 Marks]**

38. Let Q be the set of all rational numbers show that $Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$ is a field under the usual addition and multiplication. **[10 Marks]**

39. Let G be the group of non-zero complex numbers under multiplication, and let N be the set of complex numbers of absolute value 1. Show that G / N is isomorphic to group of all positive real numbers under multiplication. **[13 Marks]**

40. Let G be a group of under $2p, p$ prime show that either G is cyclic of G is generated by $\{a, b\}$ with relations $a^p = e = b^2$ and $bab = a^{-1}$ **[13 Marks]**

2010

41. Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \middle| a \in R, a \neq 0 \right\}$ Show that G is a group under matrix multiplication **[10 Marks]**

42. Let F be a field order 32. Show that the only subfields of F are F itself and $\{0,1\}$ **[10 Marks]**

43. Prove or disprove that $(\mathbb{R},+)$ and $(\mathbb{R}^+,)$ are isomorphic group where $\mathbb{R}^+$ denote the set of all positive real numbers. **[13 Marks]**

44. Show that the zero and unity are only idempotents of Z_n if $n = p^r$ where p is a prime. **[13 Marks]**

45. Let R be a Euclidean domain with Euclidean valuation d. Let n be an integer such that $d(1) + n \geq 0.$ show that the function $d_n : R - \{0\} \to S,$ where S is the set of all negative integers defined by $d_n(a) = d(a) + n$ for all $a \in R - \{0\}$ is a Euclidean valuation **[13 Marks]**

2009

46. Prove that a non-empty subset H of a group G is normal subgroup of $G \Leftrightarrow$ for all $x, y \in H,\ g \in G$, $(gx)(gy)^{-1} \in H$ **[10 Marks]**

47. If G is a finite abelian group then show that $O(a,b)$ is a divisor of l.c.m of $O(a), O(b)$ **[10 Marks]**

48. Show that $d(a) < d(ab)$, where a, b be two non-zero element of a Euclidean domain R and b is not a unit in R **[13 Marks]**

49. Show that a field an integral domain and a non-zero finite integral is a field **[13 Marks]**

50. Find the multiplicative inverse of the element $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ of the ring M of all matrices of order two over the integers. **[14 Marks]**