RAMANASRI
IAS/IFoS Institute

Dynamics, Statics & Hydro Statics

IFoS (IFS) Previous Year Questions (PYQ) from 2020 to 2009

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IAS, UPSC, IFS, IFoS, CIVIL SERVICE MAINS EXAMS MATHS OPTIONAL STUDY MATERIALS

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2020

1. If the radial and transverse velocities of a particle are proportional to each other, then prove that the path is an equiangular spiral. Further, if radial acceleration is proportional to transverse acceleration, then show that velocity of the particle varies as some power of the radius vector. **[8 Marks]**

2. A cylinder of radius r, whose axis is fixed horizontally, touches a vertical wall along a generation line. A flat beam of length l and weight rests with its extremities in contact with the wall and the cylinder, making angle of 45^0 with the vertical. Prove that the reaction of the cylinder is $\frac{W\sqrt{5}}{2}$ and pressure on the wall is $\frac{W}{2}$.

Also, prove that the ration of radius of cylinder to the length of the beam is $5+5:4\sqrt{2}$ **[8 Marks]**

3. A particle of mass 5 unit's moves in a straight line towards a center of force and the force varies inversely a s the cube of distance. Starting from rest at the point A distant 20 units from center of force O, it reaches a point B distant 'b' from O. Find the time in reaching from A to B and the velocity at B. When will the particle reach at the center? **[15 Marks]**

4. Derive intrinsic equation $x = c\log(\sec\psi + \tan\psi)$ of the common catenary, where symbols have usual meanings. Prove that the length of an endless chain, which will hang over a circular pulley of radius 'a' so as to be in contact with $\frac{2}{3}$ of the circumference of the pulley, is $a\left\{\frac{4\pi}{3}+\frac{3}{\log(2+\sqrt{3})}\right\}$ **[10 Marks]**

5. A sphere of radius 'a' and having density half of that of water, is completely immersed at the bottom of a circular cylinder of radius 'b',, which is filled with water to depth 'd'. The sphere is set free and takes up its position of equilibrium. Show that the loss of potential energy this way is $W\left(d-\frac{11}{8}a-\frac{a^3}{3b^2}\right)$, where W is the weight of the sphere. **[15 Marks]**

2019

6. A 2 meters rod has a weight of 2 N and has its centre of gravity at 120 cm from one end. At 20 cm, 100 cm and 160 cm from the same end are hung loads of 3 N, 7 N and 10 N respectively. Find the point at which the rod must be supported if it is to remain horizontal. **[8 Marks]**

7. Find the law of force for the orbit $\mathrm{r}^2 = \mathrm{a}^2\cos 2\theta$ (the pole being the centre of the force) **[15 Marks]**

8. A vessel is in the shape of a hollow hemisphere surmounted by a cone held with the axis vertical and vertex uppermost. If it is filled with a liquid so as to submerge half the axis of the cone in the liquid and height of the cone be double the radius (r) of its base, find the resultant downward thrust of the liquid on the vessel in terms of the radius of the hemisphere and density (ρ) of the liquid. **[15 Marks]**

9. A shot projected with a velocity u can just reach a certain point on the horizontal plane through the point of projection. So, in order to hit a mark h metres above the ground at the same point, if the shot is projected at the same elevation, find increase in the velocity of projection. **[15 Marks]**

2018

10. If the velocities in a simple harmonic motion at distances a, b and c from a fixed point on the straight line which is not the centre of force, are u, v and w respectively, show that the periodic time T is given by $\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ **[8 Marks]**

11. From a semi-circle whose diameter is in the surface of a liquid, a circle is cut out, whose diameter is the vertical radius of the semi-circle. Find the depth of the centre of pressure of the remainder part. **[8 Marks]**

12. Let T_1 and T_2 be the periods of vertical oscillations of two different weights suspended by an elastic string, and C_1 and C_2 are the statical extensions due to these weights and g is the acceleration due to gravity. Show that $g = \frac{4\pi^2(C_1 - C_2)}{T_1^2 - T_2^2}$ **[15 Marks]**

13. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \log \frac{1+(1+\mu^2)^{\frac{1}{2}}}{\mu}$ where μ is the coefficient of friction. **[10 Marks]**

14. A frame ABC consists of three light rods, of which AB, AC are each of length, a, BC of length, $\frac{3}{2}a$ freely jointed together. It rests with BC horizontal, A below BC and the rods AB, AC over two smooth pegs E and F in the same horizontal line, at a distance $2b$ apart. A weight W is suspended from A. Find the thrust in the rod BC. **[10 Marks]**

15. A solid hemisphere floating in a liquid is completely immersed with a point of the rim joined to a fixed point by means of a string. Find the inclination of the base to the vertical and tension of the string. **[15 Marks]**

16. A snowball of radius $r(t)$ melts at a uniform rate. If half of the mass of the snowball melts in one hour, how much time will it take for the entire mass of the snowball to melt, correct to two decimal places? Conditions remain unchanged for the entire process. **[15 Marks]**

2017

17. (c)A particle is undergoing simple harmonic motion of period T about a centre O and it passes through the position $P(OP = b)$ with velocity v in the direction OP prove that the time that elapses before it returns to P is $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$ **[8 Marks]**

18. A heavy uniform cube balance on the highest points of a sphere whose radius is r. if the sphere is rough enough to prevent sliding and if the side of the cable be $\frac{\pi r}{2}$ then proving that the total angle through which the cube can swing without falling is 90° **[8 Marks]**

19. A string of length a from the shorter diagonal of a rhombus formed of four uniform road each of length b and weight W which are hinged together If one of the road is supporter in a horizontal position then prove that the tension of the string is $\frac{2w(2b^2-a^2)}{b\sqrt{4b^2-a^2}}$. **[10 Marks]**

20. A planet is describing an ellipse about the sun as a focus show that its velocity away from the sun is the greatest when the radius vector to the planet is a right angle to the major axis of path and the velocity then is $\frac{2\pi ae}{T\sqrt{1-e^2}}$ when 2a is the major axis e is the eccentricity and T is the periodic time. **[10 Marks]**

21. A semi-ellipse bounded by its minor axis is just immersed in a liquid, the density of which varies as the depth. If the minor axis lies on the surface the find the eccentricity in order that the focus may be the centre of pressure. **[10 Marks]**

22. A particle moves in a straight line its acceleration directed towards a fixed points O in the line and is always equal to $\mu\left(\frac{a^5}{x^2}\right)^{\frac{1}{3}}$ when it is at a distance x from O if it starts from rest at a distance a from O then prove that it will arrive at O with a velocity $a\sqrt{6\mu}$ after time $\frac{8}{15}\sqrt{\frac{6}{\mu}}$. ~~**NO marks**~~

2016

23. A weight W is hanging with the help of two strings of length l and $2l$ in such a way that the other ends A and B of those strings lie on a horizontal line at a distance $2l$. Obtain the tension in the two strings **[8 Marks]**

24. From a point in a smooth horizontal plane a, particle is projected with velocity u at angle α with respect to the horizontal form the foot of a plane inclined at an angle with respect to the horizontal from the foot of a plane, inclined at an angle β with respect to the horizon. Show that it will strike the plane at right angles if $\cot\beta = 2\tan(\alpha-\beta)$ **[8 Marks]**

25. A stone is thrown vertically with the velocity which would just carry it to a height of 40m. Two seconds later another stone is project vertically from the same place with the same velocity. When and where will they meet? **[10 Marks]**

26. Water is flowing through a pipe of 80 mm diameter under a gauge pressure of 60KPa, with a mean velocity of 2/s Find the total head if the pipe7m is above the datum line. **[10 Marks]**

27. A uniform rod of weight w is resting against an equally rough horizon and a wall at an angle α with the wall. At this condition a horizontal force P is stopping them from sliding implemented at the mid-point of the rod prove that $P = W\tan(\alpha-2\lambda)$ where λ the angle of friction is. Is there any condition on λ and α ? ~~**[10 Marks]**~~

28. A body immersed in a liquid is balanced by weight p to which it is attached by a thread passing over a fixed pulley and when half immersed is balanced in the same manner by weight prove that the density of the body the liquid are in the ratio 3:2. **[10 Marks]**

29. A particle is acted on a force parallel to the axis of whose acceleration is λy initially projected with a velocity $a\sqrt{\lambda}$ parallel to x-axis at the point where $y = a$ prove that it will describe a centenary **[10 Marks]**

2015

30. A heavy particle is attached to one end of an elastic sting, the other end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natural length a and then let go. Find the time taken by the particle to return to the starting point. **[8 Marks]**

31. A cylindrical vessel on a horizontal circular base of radius a is filled with a liquid of density w with a height h. If a sphere of radius c and density greater than w is suspended by a thread so that it is completely immersed, determine the increase of the whole pressure on the curve surface. **[8 Marks]**

32. Determine the length of an endless chain which will change over a circular pulley of radius a so as to be in contact with two-third of the circumference of the pulley. **[15 Marks]**

33. A particle of mass m is falling under the influence of gravity thought a medium whose resistance equals μ times the velocity if the particle were released from rest, determine the distance fallen thought in time t. **[10 Marks]**

34. As ellipse is just immersed in water with its major axis vertical if the centre of pressure coincides with the focus determine the eccentricity of the ellipse. **[15 Marks]**

35. A particle move with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, determine the equation to its path. **[15 Marks]**

2014

36. A particle whose mass is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance 'a' from the origin prove that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$ **[8 Marks]**

37. A hollow weightless hemisphere filed with liquid is suspended from a point on the rim of its bass. Show that the ratio of the thrust on the plane bass to the weight of the contained liquid is $12:\sqrt{73}$ **[8 Marks]**

38. An engine at a constant rate H, draws a load M against a resistance R .show that the maximum speed is and time taken to attain half of this speed is $\frac{MH}{R^2}\left(\log 2-\frac{1}{2}\right)$ **[10 Marks]**

39. A semi circle disc rests in a vertical plane with its curve edge on a rough horizontal and equally rough vertical plane. If the coeff. of friction is μ, prove that the greatest angle that the bounding diameter can make with the horizontal plane is : $\sin^{-1}\left(\frac{3\pi}{4}\frac{\mu+\mu^2}{1+\mu^2}\right)$ **[15 Marks]**

40. A body floating water has volumes V_1, V_2 and V_3 above the surface when the densities of the surrounding air are p_1, p_2, p_3 respectively. Prove that $\frac{p_2-p_3}{V_1}+\frac{p_3-p_1}{V_2}+\frac{p_1-p_2}{V_3}=0$ **[10 Marks]**

2013

41. A particle is performing a simple harmonic motion of period T about centre o and it passes through a point P, where $OP=b$ with velocity V in the direction of OP Find the time which element elapses before it returns to P **[8 Marks]**

42. A triangular lamina ABC of density p floats in liquid of density σ with its plane vertical, the angle B being in the surface of the liquid, and the angle A not immersed. Find $\frac{\rho}{\sigma}$ in terms of the lengths of the sides of the triangle. **[8 Marks]**

43. A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and discuss the nature of equilibrium. **[8 Marks]**

44. Two bodies of weight W_1 and W_2 are placed on an inclined plane and are connected by a light string which coincides with a line of greatest slope of the plane; if the coefficient of friction between the bodies and the plane are respectively μ_1 and μ_2 find the inclination **[14 Marks]**

45. A body floating in water has volumes V_1, V_2 and V_3 above the surface, when the densities of the surrounding air respectively P_1, P_2, P_3. Find the value of : $\frac{p_2-p_2}{v_1}+\frac{p_3-p_1}{v_2}+\frac{p_1-p_2}{v_3}$ **[13 Marks]**

46. A particle is projected vertically upwards with a velocity, u, in a resisting medium which produces retardation kv^2 when the velocity is V Find the height when the particle comes to rest above the point of projection. **[14 Marks]**

47. A particle is projected with a velocity V along a smooth horizontal plane in a medium whose resistance per unit mass is double of the velocity. Find the distance it will describe in time t. **[13 Marks]**

2012

48. A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to infinity. Prove that the time it takes to reach height h is $\frac{1}{3}\sqrt{\left(\frac{2a}{g}\right)}\left[\left(1+\frac{h}{a}\right)^{3/2}-1\right]$ **[8 Marks]**

49. A triangle ABC is immersed in liquid with the vertex C in the surface and the sides AC. BC equally inclined to the surface. Show that the vertical C divides the triangle in to two other the fluid pressures on which are as $b^3+3ab^2 : a^3+3a^2b$ where a and b are the sides BC & AC respectively. **[8 Marks]**

50. A particle is projected with a velocity and strikes at right angle on a plane though the plane of projection inclined at an angle β to the horizon. Show that the time of flight is $\frac{2u}{g\sqrt{(1+3\sin^2\beta)}}$ range on the plane is $\frac{2u^2}{g}.\frac{\sin\beta}{1+3\sin^2\beta}$ and the vertical height of point stuck is $\frac{2u^2\sin^2\beta}{g(1+3\sin^2\beta)}$ above the point of projection. **[10 Marks]**

51. A particle is moving with central acceleration $\mu\left[r^5-c^4\right]$ being projected from an apse at a distance with c velocity $\sqrt{\left(\frac{2\mu}{3}\right)c3}$, show that its path is a curve $x^4+y^4=c^4$ **[13 Marks]**

52. A thin equilateral rectangular plate of uniform thickness and density rests with one of its base on rough horizontal plane and the other against a small vertical wall. Show that the least angle, its base can make with the horizontal plane is given by $\cot\theta=2\mu+\frac{1}{\sqrt{3}}\mu$, being the coefficient of friction. **[14 Marks]**

53. A semicircular area of radius a is immersed vertically with its diameter horizontal at a depth b. If the circumference be below the centre. Prove that depth of centre of pressure is $\frac{1}{4}\frac{3\pi(a^2+4b^2)+32ab}{4a+3\pi b}$ **[13 Marks]**

54. A heavy elastic sting, whose natural length is $.2\pi a$, is placed round a smooth cone whose axis is vertical and whose semi vertical angle is If W be the weight and λ the modulus of elasticity of the sting, prove that it will be in equilibrium when in the form of a circle whose radius is $a\left(1+\frac{W}{2\pi\lambda}\cot\alpha\right)$. **[10 Marks]**

2011

55. The apses of a satellite of the Earth are at distances r_1 and r_2 form the centre of the Earth. Find the velocities at the apses in terms of r_1 and r_2. **[10 Marks]**

56. A cable of length 160 meters and weighing 2 kg per meter is suspended from two points in the same horizontal plane. The tension at the points of support is 200kg. Show that the span of the cable is $120\cosh^{-1}\left(\frac{5}{3}\right)$ and also find the sag. **[10 Marks]**

57. One end of a uniform rod AB of length 2a and weight W, is attached by frictionless joint to a smooth wall and the other end B is smoothly hinged to an equal rod BC the middle points of the

rods are connected by an elastic cord of natural length a and modulus of elasticity 4W prove that the system can rest in equilibrium in a vertical plane with C in contact with the wall below A and the angle between the rod is $2\sin^{-1}\left(\frac{3}{4}\right)$ **[13 Marks]**

58. AB is a uniform rod of length 8a which can turn freely about the end A which is fixed C is a smooth ring whose weight is twice that of the rod, which can slide on the rod and is attached by a string CD to a point D in the same horizontal plane as the point A If AD and CD are each of length a fix the position of the ring and the tension of the string when the system is in equilibrium. Show also that the action on the rod at the fixed end A is a horizontal force equal to $\sqrt{3}$ W, where W is the weight of the rod. **[14 Marks]**

59. A steam is rushing from a boiler though a conical pipe the diameter of the ends of which are D and d; If V and v be the corresponding velocities of the stream and if motion is supposed to be that of the divergence from the vertex of cone, Prove that $\frac{v}{V}=\frac{D^2}{d^2}e^{(v^2-V^2)/2k}$ where K is pressure divided by the density and supposed constant. **[13 Marks]**

2010

60. A uniform rod AB rest with one end on a smooth vertical wall and the other on a smooth inclined plane, making an angle α with the horizontal. Find the position of equilibrium and discuss stability. **[8 Marks]**

61. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex fall on the other end of the base .If θ_1 and θ_2 be the base angle and be the angle of projection, prove that $\tan\theta=\tan\theta_1+\tan\theta_2.\ \tan\theta=\tan\theta_1+\tan\theta_2.$ **[8 Marks]**

62. Prove that the horizontal line through the centre of pressure of a rectangle immersed in a liquid with one side in the surface, divides the rectangle in two parts, the fluid pressure on which are in the radio, 4:5 **[8 Marks]**

63. A uniform chain of length A and B in the same horizontal line A load P is now hung from the middle point D of chain and the depth of this point below AB is found to be h. show that each terminal tension is, $\frac{1}{2}\left[p\frac{l}{h}+w\frac{h^2+l^2}{2hl}\right]$ **[14 Marks]**

64. A particle move with a central acceleration $\frac{\mu}{(dis\tan ce)^2}$, it is projected with velocity V at a distance R show that its path is a rectangular hyperbola if the angle of projection is,

$$\sin^{-1}\left[\frac{\mu}{VR\left(V^2-\frac{2\mu}{R}\right)^{1/2}}\right]$$ **[13 Marks]**

65. A smooth wedge of mass M is placed smooth horizontal plane and a particle of mass m sides down its slant face which is inclined at angle α to the horizontal plane prove that the acceleration of the wedge is, $\frac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha}$ **[13 Marks]**

2009

66. A uniform rectangular board, whose sides are 2a and 2b, rests in limiting equilibrium in contact with two rough pegs in the same horizontal line at a distant d apart show that the inclination θ of the side 2a to the horizontal is given by the equation $d\cos\lambda[\cos(\lambda+2\theta)]=a\cos\theta-b\sin\theta$ where λ is the angle of friction. **[10 Marks]**

67. A particle rests in equilibrium under the attraction of two centres of force which attract directly as the distance their intensities being μ and μ'. The particle is slightly displaced towards one of them, show that the time of small oscillation is $\frac{2\pi}{\sqrt{(\mu+\mu)}}$. **[10 Marks]**

68. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall which the curved surface of the sphere is in contact If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical prove that $\tan\phi=\frac{3}{8}+\tan\theta$.

[10 Marks]

69. A particle moves with a central acceleration $\mu\left(\gamma+\frac{a^4}{\gamma^3}\right)$ being projected from an apse at a distance a with a velocity $2\sqrt{\mu}a$ prove that its path is $\gamma^2(2+\cos\sqrt{3}\theta)=3a^2$ **[10 Marks]**

70. A shell, lying in a straight smooth horizontal tube, suddenly exploder and breaks into potion of masses m and m' if d, is the distance apart of the masses after a time t, show that the work done by the explosion is $\frac{1}{2}\frac{mm'}{m+m'}\frac{d}{t^2}$ **[10 Marks]**

71. A hollow conical vessel floats in water with its vertex downward and a certain depth of its axis immersed when water is poured into it up to level with the surface of the water what portion of axis was originally immersed? **[10 Marks]**