

Complex Analysis

IFoS (IFS) Previous Year
Questions (PYQ) from
2020 to 2009

Ramana sri Sir

IAS, UPSC, IFS, IFoS, CIVIL
SERVICE MAINS EXAMS
MATHS OPTIONAL STUDY
MATERIALS

Ramanasri IAS/IFoS(IFS) Maths Optional Complex Analysis PYQ 2020 to 2009

2020

1. Evaluate the integral $\int_C \operatorname{Re}(z)^2 dz$ from 0 to $2+4i$ along the curve $C: y = x^2$. [8 Marks]
2. Using Cauchy theorem and Cauchy integral formula, evaluate the integral $\oint_C \frac{e^z}{z^2(z+1)^3} dz$ where C is $|z|=2$. [15 Marks]
3. Show that the bilinear transformation $w = e^{i\theta_0} \left(\frac{z-z_0}{z-\bar{z}_0} \right)$ z_0 being in the upper half of the z -plane, maps the upper half of the z -plane into the interior of the unit circle in the w -plane. If under this transformation, the point $z=i$ is mapped into $w=0$ while the point at infinity is mapped into $w=-1$, then find this transformation. [10 Marks]

2019

4. Using Cauchy's Integral formula, evaluate the integral $\oint_C \frac{dz}{(z^2+4)^2}$ where $c: |z-i|=2$. [8 Marks]
5. If $f(z)$ is analytic in a domain D and $|f(z)|$ is a non-zero constant in D , then show that $f(z)$ is constant in D . [15 Marks]
6. Classify the singular point $z=0$ of the function $f(z) = \frac{e^z}{z + \sin z}$ and obtain the principal part of the Laurent series expansion of $f(z)$. [15 Marks]

2018

7. If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine v so that $u+iv$ is a regular function of $x+iy$. [10 Marks]
8. Evaluate the integral $\int_0^{2\pi} \cos^2 \theta d\theta$, where n is a positive integer. [10 Marks]

2017

9. If $f(z) = u(x, y) + iv(x, y)$ is an analytic function of $z = x + iy$ and $u + 2v = x^3 - 2y^3 + 3xy(2x - y)$ then find $f(z)$ in terms of z . [8 Marks]
10. Prove by the method of contour integration that $\int_0^\pi \frac{1 + 2\cos \theta}{5 + 4\cos \theta} d\theta = 0$. [12 Marks]
11. Find the sum of residues of $f(z) = \frac{\sin z}{\cos z}$ at its poles inside the circle $|z|=2$. [8 Marks]

2016

12. Find the analytic function of which the real part is $e^{-x} \{(x^2 - y^2) \cos y + 2xy \sin y\}$ [8 Marks]

Ramanasri IAS/IFoS(IFS) Maths Optional Complex Analysis PYQ 2020 to 2009

13. Find the Laurent series for the function $f(z) = \frac{1}{1-z^2}$ with center $z = 1$ [10 Marks]

14. Evaluate by contour integration $\int_0^\pi \frac{d\theta}{\left(1 + \frac{1}{2} \cos \theta\right)^2}$ [10 Marks]

2015

15. Let $u(x, y) = \cos x \sinh y$. Find the harmonic conjugate $v(x, y)$ of u and express $u(x, y) + iv(x, y)$ as a function of $z = x + iy$. [12 Marks]

16. Evaluate the integral $\int_r \frac{z^2}{(z^2 + 1)(z - 1)^2} dz$, where r is the circle $|z| = 2$. [12 Marks]

17. Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$ by using contour integration and the residue theorem. [15 Marks]

2014

18. Using Cauchy integral formula, Evaluate $\int_C \frac{z+2}{(z+1)^2(z-2)} dz$ where C is the circle $|z-i|=2$. [8 Marks]

19. Find the constants a, b, c such that the function $f(z) = 2x^2 - 2xy - y^2 + i(ax^2 - bxy + cy^2)$ is analytic for all $z (= x + iy)$ and express $f(z)$ in terms of z . [8 Marks]

20. Evaluate $\int_{|z|=1} \frac{z}{z^4 - 6z^2 + 1} dz$ [15 Marks]

21. Find the bilinear transformation which map the point $-1, \infty, i$ into the points-
(i) $i, 1, 1+i$
(ii) $\infty, i, 1$
(iii) $0, \infty, 1$ [15 Marks]

22. Find the Laurent series expansion at $z = 0$ for the function $f(z) = \frac{1}{z^2(z^2 + 2z - 3)}$ in the region
(i) $1 < |z| < 3$ And (ii) $|z| > 3$. [15 Marks]

2013

23. Construct an analytic function $f(z) = u(x, y) + iv(x, y)$, where $v(x, y) = 6xy - 5x + 3$ Express the result as a function of z [10 Marks]

24. Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 3$ [13 Marks]

Ramanasri IAS/IFoS(IFS) Maths Optional Complex Analysis PYQ 2020 to 2009

25. Find Laurent series about the indicated singularity. Name the singularity and the region of convergence $\frac{z - \sin z}{z^3}; z = 0$ [13 Marks]

2012

26. Evaluate the integral $\int_{2-i}^{4+i} (x + y^2 - ixy) dz$ along the line segment AB joining the point $A(2, -1)$ and $B(4, 1)$. [10 Marks]
27. Show that the function $u(x, y) = e^{-x}(x \cos y + y \sin y)$ is harmonic. Find its conjugate harmonic function $v(x, y)$ and the corresponding analytic function $f(z)$. [13 Marks]
28. Using the Residue Theorem, evaluate the integral $\int_C \frac{e^z - 1}{z(z-1)(z+i)^2} dz$, where C is the circle $|z| = 2$ [13 Marks]

2011

29. Expand the function $f(z) = \frac{2z^2 + 11z}{(z+1)(z+4)}$ in a Laurent's series valid for $2 < |z| < 3$. [10 Marks]
30. States Cauchy's residue theorem Using it, evaluate the integral $\int_C \frac{e^{z/2}}{(z+2)(z^2-4)} dz$ counterclockwise around the circle $C: |z+1| = 4$ [13 Marks]

2010

31. Determine the analytic function $f(z) = u + iv$. if $v = e^x(x \sin y + y \cos y)$ [10 Marks]
32. Using the method of contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)^2(x^2 + 2x + 2)}$ [14 Marks]
33. Obtain Laurent's series expansion of the function $f(z) = \frac{1}{(z+1)(z+3)}$ in the region $0 < |z+1| < 2$ [13 Marks]

2009

34. Show that under the transformation $w = \frac{z-i}{z+i}$ real axis in the z -plane is mapped into the circle $|w| = 1$. what portion of the z -plane corresponds to the interior of the circle? [10 Marks]
35. Evaluate $\int_C \frac{2z+1}{z^2+z} dz$ by Cauchy's integral formula, where C is $|z| = \frac{1}{2}$ [10 Marks]

Ramanasri IAS/IFoS(IFS) Maths Optional Complex Analysis PYQ 2020 to 2009

36. Determine the analytic function $w = u + iv$, if $u = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$ [13 Marks]
37. Evaluate by contour integration $\int_0^{2\pi} \frac{d\theta}{1 - 2a \sin \theta + a^2}, 0 < a < 1$ [13 Marks]

RAMANASRI