RAMANASRI
IAS/IFoS Institute

Analytical Geometry

Indian Forest Service (IFoS) Maths Optional Previous Year Questions (PYQ) from 2020 to 2009

IAS, UPSC, CIVIL SERVICES, IFoS MAINS EXAMS MATHS OPTIONAL STUDY MATERIALS

WEBSITE: MATHEMATICSOPTIONAL.COM
CONTACT: 8750706262

Ramanasri IAS/IFoS Maths Optional Analytical Geometry PYQ 2020 to 2009

2020

1. If the straight lines, joining the origin to the points of intersection of the curve $3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0$ and the straight line $2x + 3y + k = 0$ are at right angles, then show that $6k^2 + 5k + 52 = 0$ **[8 Marks]**

2. Prove that the angle between two straight whose direction cosines are given by $l + m + n = 0$ and $fmn + gnl + hlm = 0$ is $\frac{\pi}{3}$ if $\frac{1}{f} + \frac{1}{g} + \frac{1}{h} = 0$ **[15 Marks]**

3. A point P moves on the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ which is fixed. The plane through P and perpendicular to OP meets the axes in A, B, C respectively. The planes through A, B, C parallel to yz, zx and xy planes respectively intersect at Q. Prove that the locus of Q is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}$. **[15 Marks]**

4. Let P be the vertex of the enveloping cone of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. If the section of this cone made by the plane $z = 0$ is a rectangular hyperbola, then find the locus of P **[10 Marks]**

2019

5. Find the volume lying inside the cylinder $\mathrm{x}^2 + \mathrm{y}^2 - 2\mathrm{x} = 0$ and outside the paraboloid $\mathrm{x}^2 + \mathrm{y}^2 = 2\mathrm{z}$, while bounded by xy-plane. **[8 Marks]**

6. If the coordinates of the points A and B are respectively $(\mathrm{b}\cos\alpha, \mathrm{b}\sin\alpha)$ and $(\mathrm{a}\cos\beta, \mathrm{a}\sin\beta)$ and if the line joining A and B is produced to the point $\mathrm{M}(\mathrm{x}, \mathrm{y})$ so that $\mathrm{AM} : \mathrm{MB} = \mathrm{b} : \mathrm{a}$, then show that $\mathrm{x}\cos\frac{\alpha+\beta}{2} + \mathrm{y}\sin\frac{\alpha+\beta}{2} = 0.$ **[8 Marks]**

7. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$ **[15 Marks]**

8. Show that the shortest distance between the straight lines $\frac{\mathrm{x}-3}{3} = \frac{\mathrm{y}-8}{-1} = \frac{\mathrm{z}-3}{1}$ And $\frac{\mathrm{x}+3}{-2} = \frac{\mathrm{y}+7}{2} = \frac{\mathrm{z}-6}{4}$ is $3\sqrt{30}$ .Find also the equation of the line of shortest distance. **[15 Marks]**

9. A variable plane is parallel to the plane $\frac{\mathrm{x}}{\mathrm{a}} + \frac{\mathrm{y}}{\mathrm{b}} + \frac{\mathrm{z}}{\mathrm{c}} = 0$ and meets the axes at the points A, B and C. Prove that the circle ABC lies on the cone $\mathrm{yz}\left(\frac{\mathrm{b}}{\mathrm{c}} + \frac{\mathrm{c}}{\mathrm{b}}\right) + \mathrm{zx}\left(\frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{a}}{\mathrm{c}}\right) + \mathrm{xy}\left(\frac{\mathrm{a}}{\mathrm{b}} + \frac{\mathrm{b}}{\mathrm{a}}\right) = 0$ **[15 Marks]**

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2018

10. Find the equations of the tangent planes to the ellipsoid $2x^2+6y^2+3z^2=27$ which pass through the line $x-y-z=0=x-y+2z-9$ **[8 Marks]**

11. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=4, z=2$ **[10 Marks]**

12. Find the equation of the tangent plane that can be drawn to the sphere $x^2+y^2+z^2-2x+6y+2z+8=0$ through the straight line $3x-4y-8=0, y-3z+2$ **[10 Marks]**

13. Find the equations of the straight lines in which the plane $2x+y-z=0$ cuts the cone $4x^2-y^2+3z^2=0$ Find the angle between the two straight lines. **[10 Marks]**

14. Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ **[10 Marks]**

2017

15. Find the equation of the planes parallel to the plane $3x-2y+6z+8=0$ and at a distance 2 from it. **[8 Marks]**

16. Show that the angle between the plane given by the equation $2x^2-y^2+3z^2-xy+7zx+2yz=0, \tan^{-1}\frac{\sqrt{50}}{4}$ **[10 Marks]**

17. Find the angle between line whose direction cosines are given by the relation $l+m+n=0$ and $2lm+2\ln-mn=0$ **[10 Marks]**

18. Find the equation of the circular cone with vectors at the origin and whose axis makes equal angle with the coordinate axes and the generator is the line passing through the origin with direction rations (1,-2,2) **[10 Marks]**

19. Fine the shortest distance and the equation of the line of the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ **[10 Marks]**

2016

20. A perpendicular is drawn from the center of ellipse $\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$ to any tangent. Prove that the locus of the food of the perpendicular is given by $(x^2+y^2)^2=a^2x^2+b^2y^2$ **[10 Marks]**

21. Obtain the equation of the sphere on which the intersection of the plane $5x-2y+4z+7=0$ with the sphere which has $(0,1,0)$ and $(3,-5,2)$ as the end points of its diameter is a great circle. **[10 Marks]**

22. A plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate plane at A,B,C Find the equation of the cone with vertex at origin and guiding curve at the circle passing thought A,B,C **[10 Marks]**

2015

23. Find the equation of the plane containing the straight line $y+z=1,\ x=0$ and parallel to the straight the line $x-z=1,\ y=0.$ **[10 Marks]**

24. Find the locus of the variable straight line the always intersects **[10 Marks]**
$x=1,y=0;y=1,z=0;z=1,x=0.$

25. Find the locus of the poles of chords which are normal to the parabola $y^2=4ax.$ **[10 Marks]**

26. The tangent at $(a\cos\theta,b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the auxiliary circle in two points. The chord joining them subtends a right angle at the center find the eccentricity of the ellipse. **[8 Marks]**

2014

27. Prove that the locus of a variable line which intersects the three lines
$y=mx,z=c;y=-mx,z=-c;y=z,mx=-c$ is the surface $y^2-m^2x^2=z^2-c^2$ **[8 Marks]**

28. Prove that every sphere passing through the circle $x^2+y^2-2ax+r^2=0,z=0$ cut orthogonally every sphere though the circle $x^2+z^2=r^2,y=0$ **[10 Marks]**

29. A moving plane passes through a fixed point $(2,2,2)$ and meets the coordinate axes at the points $A,B,C,$ plane all away from the origin $O.$ Find the locus of the centre of the sphere passing though the points O,A,B,C **[10 Marks]**

30. Prove that the equation $4x^2-y^2+z^2-3yz+12x-11y+6z+4=0$ represents a cone with vertex at $(-1,2,-3)$ **[10 Marks]**

31. Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ **[10 Marks]**

2013

32. Find the surface generated by the straight line which intersects the lines $y=z=a$ and $x+3z=a=y+z$ and is parallel to the plane $x+y=0.$ **[8 Marks]**

33. Reduce the following equation to its canonical from and determine the nature of the conic
$4x^2+4xy+y^2-12x-6y+5=0.$ **[10 Marks]**

34. Find the equation to the tangent plane to the surface $7x^2 - 3y^2 - z^2 + 21 = 0,$ which Pass though the line $7x - 6y + 9 = 0, z = 3.$ **[10 Marks]**

35. Find the magnitude and the equation of the line of shortest distance between the line

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$ **[10 Marks]**

2012

36. Find the equation to the lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$ **[8 Marks]**

37. Find the equation of the tangent plane to the ellipsoid $2x^2 + 6y^2 + 3z^2 = 27$ which passes through the line $x - y - z = 0 = x - y + 2z - 9$ **[10 Marks]**

38. If $2C$ is the shortest distance between the lines $\frac{x}{i} - \frac{z}{n} = 1, y = 0$ and $\frac{y}{m} + \frac{z}{m} = 1, x = 0$ then show that

$$\frac{1}{i^2} + \frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{c^2}.$$ **[10 Marks]**

39. Show that all the sphere, that can be drawn though the origin and each set of point where planes parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ cut the ordinate axes, form a system of sphere which are cut orthogonally by the sphere $x^2 + y^2 + 2fx + 2gy + 2hz = 0$ if $af + bg + ch = 0.$ **[10 Marks]**

40. A plane makes equal intercepts on the positive parts of the axes and touches the ellipsoid $x^2 + 4y^2 + 9z^2 = 36.$ Find its equation. **[10 Marks]**

2011

41. A variable plane is it a constant distant P from the origin and meet the axes A, B, C at prove that the locus of the centroid of the tetrahedron $OABC$ is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}.$ **[10 Marks]**

42. Find the equation of the right circle cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z=3}{2}$ **[10 Marks]**

43. Find the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which are parallel to the Plane $lx + my + nz = 0$ **[10 Marks]**

44. Prove that the Semi-latusrectum of any conic is a harmonic mean between the segments of any focal chord. **[8 Marks]**

45. Tangent planes at two-points P and Q of a parabolic meet in the line RS show that the plane though RS and middle point of PQ is parallel to the axis of the paraboloid. **[12 Marks]**

2010

46. If a plane cuts the axes in A,B,C and (a,b,c) are the coordinates of the centroid of the triangle ABC, then show that the equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3.$ **[8 Marks]**

47. Find the equation of the sphere passing through the circle $x^2+y^2+z^2-6x-2z+5=0.y=0$ and touching the plane $3y+4z+5=0.$ **[8 Marks]**

48. Prove that the second-degree equation $x^2-2y^2+3x^2+5yz-6zx-4xy+8x-19y-2z-20=0$ represents a cone whose vertex is $(1,-2,3)$ **[10 Marks]**

49. If the feet of three normal drawn from point p to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie in the Plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1,$ prove that the feet of the other three normals lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}+1=0$ **[10 Marks]**

50. If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of the three mutually perpendicular generators of the cone $5yz-8zx-3xy=0,$ find the equation of the other two. **[10 Marks]**

51. Prove that the locus of the point of intersection of three tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ which are parallel to the conjugate diametral plane of the ellipsoid $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=1$ is $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}+\frac{c^2}{\gamma^2}$ **[10 Marks]**

2009

52. Show that the plane $x+2y-z=4$ cuts the sphere $x^2+y^2+z^2-x+z=2$ in a circle of radius unity and find the equation of the sphere which has this circle as one of its great circles. **[10 Marks]**

53. Obtain the equation of the planes which pass though the point $(3,0,3)$ touch the sphere $x^2+y^2+z^2=9$ and are parallel to the line $x=2y=-z$ **[10 Marks]**

54. The section of a cone whose vertex is P and guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,z=0$ by the plane $x=0$ is a rectangular hyperbola. Show that the locus of P is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1.$ **[10 Marks]**

55. Prove that the locus of the poles of the tangent planes of the Conicoid $ax^2+by^2+cz^2=1$ with respect to the Conicoid $\alpha x^2+\beta y^2+\gamma z^2=1$ is the Conicoid $\frac{\alpha^2x^2}{a}+\frac{\beta^2y^2}{b}+\frac{\gamma^2z^2}{c}=1$ **[10 Marks]**

56. Show that the liner drawn from the origin parallel to the number to the central Conicoid $ax^2 + by^2 + cz^2 = 1$ its point of intersection with the planes $lx + my + nz = p$ generate the cone

$$p^2\left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c}\right)^2$$

[10 Marks]