Indian Forest Service Examination-2012

D–GT–M–NUB

MATHEMATICS

Paper—II

Time Allowed : Three Hours | *Maximum Marks : 200*

INSTRUCTIONS

*Candidates should attempt Question Nos. 1 and 5 which are compulsory, and any **THREE** of the remaining questions, selecting at least **ONE** question from each Section.*

All questions carry equal marks.

The number of marks carried by each part of a question is indicated against each.

*Answers must be written in **ENGLISH** only.*

Assume suitable data, if considered necessary, and indicate the same clearly.

Symbols and notations have their usual meanings, unless indicated otherwise.

Important Note :—

All parts/sub-parts of a question must be attempted contiguously. That is, candidates must complete attempting all parts/sub-parts of a question being answered in the answer book before moving on to the next question.

Pages left blank, if any, in the answer-book(s) must be clearly struck out. Answers that follow pages left blank may not be given credit.

SECTION—A

1. Answer the following :

(a) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1 & , \quad x \text{ is irrational} \\ -1 & , \quad x \text{ is rational} \end{cases}$$

is discontinuous at every point in $\mathbb{R}$. 10

(b) Show that every field is without zero divisor. 10

(c) Evaluate the integral

$$\int_{2-i}^{4+i} (x + y^2 - ixy)\, dz$$

along the line segment AB joining the points A(2, −1) and B(4, 1). 10

(d) Show that the functions :

$$u = x^2 + y^2 + z^2$$

$$v = x + y + z$$

$$w = yz + zx + xy$$

are not independent of one another. 10

2. (a) Show that in a symmetric group S_3, there are four elements σ satisfying σ^2 = Identity and three elements satisfying σ^3 = Identity. 13

(b) If

$$u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right),$$

show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2u.$$ 13

(c) Solve the following problem by Simplex Method. How does the optimal table indicate that the optimal solution obtained is not unique ?

Maximize $z = 8x_1 + 7x_2 - 2x_3$

subject to the constraints

$$x_1 + 2x_2 + 2x_3 \le 12$$

$$2x_1 + x_2 - 2x_3 \le 12$$

$$x_1, x_2, x_3 \ge 0$$ 14

3. (a) Find the volume of the solid bounded above by the parabolic cylinder $z = 4 - y^2$ and bounded below by the elliptic paraboloid $z = x^2 + 3y^2$. 13

(b) Show that the function

$$u(x, y) = e^{-x}(x \cos y + y \sin y)$$

is harmonic. Find its conjugate harmonic function $v(x, y)$ and the corresponding analytic function $f(z)$. 13

(c) If R is an integral domain, show that the polynomial ring $R[x]$ is also an integral domain. 14

4. (a) Using the Residue Theorem, evaluate the integral

$$\int_C \frac{e^z - 1}{z(z-1)(z+i)^2}\, dz,$$

where C is the circle $|z| = 2$. 13

(b) Examine the series

$$\sum_{n=1}^{\infty} u_n(x) = \sum_{n=1}^{\infty}\left[\frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2}\right]$$

for uniform convergence. Also, with the help of this example, show that the condition of uniform convergence of $\sum_{n=1}^{\infty} u_n(x)$ is sufficient but not necessary for the sum $S(x)$ of the series to be continuous. 13

(c) Find the initial basic feasible solution of the following minimum cost transportation problem by Least Cost (Matrix Minima) Method and using it find the *optimal* transportation cost :—

Destinations

		D_1	D_2	D_3	D_4	Supply
Sources	S_1	5	11	12	13	10
	S_2	8	12	7	8	30
	S_3	12	7	15	6	35
Requirement		15	15	20	25	

14

SECTION—B

5. Answer the following :

(a) Using Lagrange's interpolation formula, show that $32f(1) = -3f(-4) + 10f(-2) + 30f(2) - 5f(4)$.

10

(b) Solve

$$(D^3D'^2 + D^2D'^3)z = 0,$$

where D stands for $\frac{\partial}{\partial x}$ and D' stands for $\frac{\partial}{\partial y}$.

10

(c) Write a computer program to implement trapezoidal rule to evaluate

$$\int_0^{10} \left(1-e^{-\frac{x}{2}}\right) dx.$$ 10

(d) Prove that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = \left(\vec{\Omega}\cdot\nabla\right)\vec{q} + \nu\ \nabla^2\vec{\Omega},$$

where ν is kinematic viscosity. 10

6. (a) Using Method of Separation of Variables, solve Laplace Equation in three dimensions. 13

(b) Derive the differential equation of motion for a spherical pendulum. 13

(c) A river is 80 meters wide. The depth d (in meters) of the river at a distance x from one bank of the river is given by the following table :

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of cross-section of the river. 14

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7. (a) Show that

$$u = \frac{A(x^2 - y^2)}{(x^2 + y^2)^2}, \; v = \frac{2Axy}{(x^2 + y^2)^2}, \quad w = 0$$

are components of a possible velocity vector for inviscid incompressible fluid flow. Determine the pressure associated with this velocity field. 13

(b) Solve the following system of equations using Gauss-Seidel Method :

$$28x + 4y - z = 32$$

$$2x + 17y + 4z = 35$$

$$x + 3y + 10z = 24$$

correct to three decimal places. 13

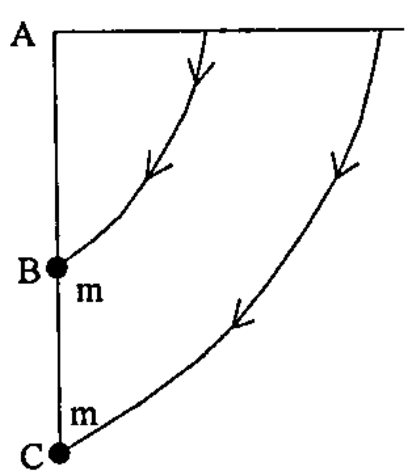
(c) Draw a flow chart for interpolation using Newton's forward difference formula. 14

8. (a) Solve

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

using Lagrange's Method. 13

(b) A weightless rod ABC of length 2a is movable about the end A which is fixed and carries two particles of mass m each one attached to the mid-point B of the rod and the other attached to the end C of the rod. If the rod is held in the horizontal position and released from rest and allowed to move, show that the angular velocity of the rod when it is vertical is $\sqrt{\frac{6g}{5a}}$. 13



(c) Using Euler's Modified Method, obtain the solution of

$$\frac{dy}{dx} = x + \left|\sqrt{y}\right|, \quad y(0) = 1$$

for the range $0 \leq x \leq 0.6$ and step size 0.2. 14