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UPSC MATHS OPTIONAL COACHING

Numerical Analysis & Computer Programming

Previous year Questions from 2020 to 1992

2021-22

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2020

1. Show that the equation $f(x) \equiv \cos\frac{\pi(x+1)}{8} + 0.148x - 0.9062 = 0$ has one root in the interval $(-1, 0)$ and one in $(0,1)$. Calculate the negative root correct to four decimal places using Newton-Raphson Method. **[10 Marks]**

2. Let $g(w,x,y,z) = (w+x+y)(x+\overline{y}+z)(w+\overline{y})$ be a Boolean function. Obtain the conjunctive normal form for $g(w,x,y,z)$. Also express $g(w,x,y,z)$ as product of maxterms. **[10 Marks]**

3. For the solution of system of equations:

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

Set up the Gauss-Seidel iterative scheme and iterate three times starting with the initial vector $X^{(0)} = 0$. Also find the exact solutions and compare with the iterated solutions. **[15 Marks]**

4. Find a quadrature formula $\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$ which is exact for polynomials of highest possible degree. Then use the formula to evaluate $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ (correct up to three decimal places). **[20 Marks]**

5. Write the three-point Lagrangian interpolating polynomial relative to the points $x_0, x_0+\varepsilon$ and x_1. Then by taking the limit $\varepsilon \to 0$, establish the relation

$$f(x) = \frac{(x_1-x)(x+x_1-2x_0)}{(x_1-x_0)^2}f(x_0) + \frac{(x-x_0)(x_1-x)}{(x_1-x_0)}f'(x_0) + \frac{(x-x_0)^2}{(x_1-x_0)}f(x_1) + E(x)$$

where $E(x) = \frac{1}{6}(x_1-x_0)^2(x-x_1)f'''(\xi)$ is the error function and

$\min(x_0, x_0+\varepsilon, x_1) < \xi < \max(x_0, x_0+\varepsilon, x_1)$. **[15 Marks]**

2019

6. Apply Newton-Raphson method, to find real root of transcendental equation, $x\log_{10} x = 1.2$ correct to three decimal places. **[10 Marks]**

7. Using Runge-Kutta method of forth order to solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$ at $x=0.2$. Use four decimal places for calculation and step length 0.2 **[10 Marks]**

8. Draw a flow chart and write a basic algorithm for (in FORTRAN/C/C^{++}) for evaluating $y = \int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule **[10 Marks]**

9. Find the equivalent numbers given in a specified number to the system mentioned against them:
(i) Integer 524 in binary system.
(ii) 101010110101. 101101011 to octal system.
(iii) decimal number 5280 to hexadecimal system.

10. (iv) Find the unknown number $(1101.101)_8 \to (?)_{10}$. **[15 Marks]**

11. Apply Gauss-Seidel iteration method to solve the following system of equations: $2x+y-2z=17$ $3x+20y-z=18$ $2x-3y+20z=25$, correct to three decimal places. **[15 Marks]**

12. Given the Boolean expression. $X = AB + ABC + A\overline{B}\overline{C} + \overline{A}\overline{C}$

(i) Draw the logical diagram for the expression.

(ii) Minimize the expression.

(iii) Draw the logical diagram for the reduced expression. **[15 Marks]**

2018

13. Using Newton's forward difference formula find the lowest degree polynomial u_x when it is given that $u_1 = 1, u_2 = 9, u_3 = 25, u_4 = 55$ and $u_2 = 105$. **[10 Marks]**

14.

15. Starting from rest in the beginning, the speed (in km/h) of a train at different times (in minutes) is given by the below table:

Time(Minutes)	2	4	6	8	10	12	14	16	18	20
Speed(Km/h)	10	18	25	29	32	20	11	5	2	8.5

Using Simpsons' $\frac{1}{3}rd$ rule, Find the approximate distance travelled (in km) in 20 minutes from the beginning. **[10 Marks]**

16. Write down the basic algorithm for solving the equation $xe^x - 1 = 0$ by bisection method, correct to 4 decimal places. **[10 Marks]**

17. Find the equivalent of numbers given in a specified number system to the system mentioned against them. **[15 Marks]**

(i) $(111011\cdot101)_2$ to decimal system

(ii) $(1000111110000\cdot00101100)_2$ to hexadecimal system

(iii) $(C4F2)_{16}$ to decimal system

(iv) $(418)_{10}$ to binary system

18. Simplify the Boolean expression: $(a+b).(\overline{b}+c)+b.(\overline{a}+\overline{c})$ By using the laws of Boolean algebra. From its truth table write it in min-terms normal form. **[15 Marks]**

19. Find the values of the constants a, b, c such that the quadrature formula

$$\int_o^h f(x)dx = h\left[af(o) + bf\left(\frac{h}{3}\right) + cf(h)\right]$$ is exact for polynomials of as high degree as possible, and hence find the order of the truncation error. **[15 Marks]**

2017

20. Explain the main steps of the Gauss-Jordan method and apply this method to find the inverse of the matrix $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$. **[10 Marks]**

21. Write the Boolean expression $z(y+z)(x+y+z)$ in the simplest form using Boolean postulate rules. Mention the rules used during simplification. Verify your result by constructing the truth table for the given expression and for its simplest form. **[10 Marks]**

22. For given equidistant values u_{-1}, u_0, u_1 and u_2 a values are interpolated by Lagrange's formula. Show that it may be written in the form $u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_0$, where $x + y = 1$. **[15 Marks]**

23. Derive the formula $\int_a^b y dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + ... + y_{n-1}) + 2(y_3 + y_6 + y_{n-3})]$. Is there any restriction on n? State that condition. What is the error bounded in the case of Simpson's $\frac{3}{8}$ rule? **[20 Marks]**

24. Write an algorithm in the form of a flow chart for Newton-Raphson method. Describe the cases of failure of this method. **[15 Marks]**

2016

25. Convert the following decimal numbers to univalent binary and hexadecimal numbers:
[i] 4096 [ii] 0.4375 [iii] 2048.0625 **[10 marks]**

26. Let $f(x) = e^{2x}\cos 3x$ for $x \in [0,1]$. Estimate the value of $f(0.5)$ Using Lagrange interpolating polynomial of degree 3 over the nodes $x = 0, x = 0.3, x = 0.6$ and $x = 1$. Also compute the error bound over the interval $[0,1]$ and the actual error $E(0.5)$ **[20 marks]**

27. For an integral $\int_{-1}^{1} f(x)dx$ show that the two-point Gauss quadrature rule is given by $\int_{-1}^{1} f(x)dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)$ using this rule estimate $\int_2^4 2xe^x dx$ **[15 marks]**

28. Let A, B, C be Boolean variable denote complement $A, A + B$ of is an expression for $A\ OR\ B$ and $B.A$ is an expression for $AANDB$. Then simplify the following expression and draw a block diagram of the simplified expression using AND and OR gates.
$A.(A + BC).(\overline{A} + B + C).(A + \overline{B} + C).(A + B + \overline{C}).$ **[15 marks]**

2015

29. Find the principal [or canonical] disjunctive normal form in three variables p, q, r for the Boolean expression $\left((p \wedge q) \to r\right) \vee \left((p \wedge q) \to -r\right)$. Is the given Boolean expression a contradiction or a tautology? **[10 Marks]**

30. Find the Lagrange interpolating polynomial that fits the following data:

x	-1	2	3	4
$f(x)$	-1	11	31	69

Find $f(1.5)$ **[20 Marks]**

31. Solve the initial value problem $\frac{dy}{dx} = x(y - x),\ y(2) = 3$ in the interval $[2,\ 2.4]$ using the Runge-Kutta fourth-order method with step size $h = 0.2$ **[15 Marks]**

32. Find the solution of the system

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

using Gauss-Seidel method (make four iterations) **[15 Marks]**

2014

33. Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^x = 0$ correct up to four decimal places. **[10 Marks]**

34. Use five subintervals to integrate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule. **[10 Marks]**

35. Use only AND and OR logic gates to construct a logic circuit for the Boolean expression $z = xy + uv$ **[10 Marks]**

36. Solve the system of equations

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1$$

using Gauss-Seidel iteration method **[**perform three iterations**]** **[15 Marks]**

37. Use Runge-Kutta formula of fourth order to find the value of y at $x = 0.8$, where $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$. Take the step length $h = 0.2$ **[20 Marks]**

38. Draw a flowchart for Simpson's one-third rule. **[15 Marks]**

39. For any Boolean variables x and y, show that $x + xy = x$. **[15 Marks]**

2013

40. In an examination, the number of students who obtained marks between certain limits were given in the following table:

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

Using Newton's forward interpolation formula, find the number of students whose marks lie between 45 and 50. **[10 Marks]**

41. Develop an algorithm for Newton-Raphson method to solve $f(x) = 0$ starting with initial iterate x_0, n be the number of iterations allowed, epsilon be the prescribed relative error and delta be the prescribed lower bound for $f'(x)$ **[20 Marks]**

42. Use Euler's method with step size $h = 0.15$ to compute the approximate value of $y(0.6)$, correct up to five decimal places from the initial value problem. $y' = x(y+x) - 1$, $y(0) = 2$ **[15 Marks]**

43. The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using composite Simpson's $\frac{1}{3}$ rule. **[15 Marks]**

2012

44. Use Newton-Raphson method to find the real root of the equation $3x = \cos x + 1$ correct to four decimal places **[12 Marks]**

45. Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ in the interval $[a,b]$ for n number of discrete points, where the initial value is $y(a) = \alpha,$ using Euler's method. **[15 Marks]**

46. Solve the following system of simultaneous equations, using Gauss-Seidel iterative method :

$$3x + 20y - z = -18$$
$$20x + y - 2z = 17$$
$$2x - 3y + 20z = 25$$

[20 Marks]

47. Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data:

$x:$	0.1	0.2	0.3	0.4
$y:$	0.9975	0.9900	0.9776	0.9604

[20 Marks]

48. In a certain examination, a candidate has to appear for one major & two major subjects .The rules for declaration of results are marks for major are denoted by M_1 and for minors by M_2 and M_3.If the candidate obtains 75% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class with distinction. If the candidate obtains 60% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class. If the candidate obtains 50% or above in major, 40% or above in each of the two minors and an average of 50% or above in all the three subjects put together, the candidate is declared to have passed the examination in second class. All those candidates, who have obtained 50% and above in major and 40% or above in minor, are declared to have passed the examination. If the candidate obtains less than 50% in major or less than 40% in anyone of the two minors, the candidate is declared to have failed in the examinations. Draw a flow chart to declare the results for the above. **[20 Marks]**

2011

49. Calculate $\int_2^{10} \frac{dx}{1+x}$ **[**up to 3 places of decimal**]** by dividing the range into 8 equal parts by Simpson's $\frac{1}{3}$ rd rule. **[12 Marks]**

50. [i] Compute $(3205)_{10}$ to the base 8.

[ii] Let A be an arbitrary but fixed Boolean algebra with operations $\wedge, \vee$ and ' and the zero and the unit element denoted by 0 and 1 respectively. Let $x, y, z...$ be elements of A. If $x, y \in A$ be such that $x \wedge y = 0$ and $x \vee y = 1$ then prove that $y = x'...$ **[12 Marks]**

51. A solid of revolution is formed by rotating about the x- axis, the area between the x- axis, the line $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates:

x	0.00	0.25	0.50	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Find the volume of the solid. **[20 Marks]**

52. Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit:

x	y	z	$f(x,y,z)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

[20 Marks]

53. Draw a flow chart for Lagrange's interpolation formula. **[20 Marks]**

2010

54. Find the positive root of the equation $10xe^{-x^2} - 1 = 0$ correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. **[12 Marks]**

55. [i] Suppose a computer spends 60 per cent of its time handling a particular type of computation when running a given program and its manufacturers make a change that improves its performance on that type of computation by a factor of 10. If the program takes 100 sec to execute, what will its execution time be after the change?

    [ii] If $A \oplus B = AB' + A'B$, find the value of $x \oplus y \oplus z$. **[6+6=12 Marks]**

56. Given the system of equations

$$2x + 3y = 1$$
$$2x + 4y + z = 2$$
$$2y + 6z + Aw = 4$$
$$4z + Bw = C$$

State the solvability and uniqueness conditions for the system. Give the solution when it exists.

[20 Marks]

57. Find the value of the integral $\int_1^5 \log_{10} x \, dx$ by using Simpson's $\frac{1}{3}$ rd rule correct up to 4 decimal places. Take 8 subintervals in your computation. **[20 Marks]**

58. [i] Find the hexadecimal equivalent of the decimal number $(587632)_{10}$

    [ii] For the given set of data points $(x_1, f(x_1), (x_2, f(x_2), \ldots (x_n, f(x_n)$ write an algorithm to find the value of $f(x)$ by using Lagrange's interpolation formula

    [iii] Using Boolean algebra, simplify the following expressions

    [a] $a + a'b + a'b'c + a'b'c'd + \ldots$

    [b] $x'y'z + yz + xz$ where x' represents the complement of x **[5+10+5=15 Marks]**

2009

59. [i] The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the iterative method given by: $x_{k+1} = -\dfrac{(ax_k + b)}{x_k}, k = 0,1,2\ldots$ is convergent near $x = \alpha$, if $|\alpha| > |\beta|$

[ii] Find the values of two valued Boolean variables A, B, C, D by solving the following simultaneous equations:

$$\bar{A} + AB = 0$$
$$AB + AC$$
$$AB + A\bar{C} + CD = \bar{C}D$$

where $\bar{x}$ represents the complement of x **[6+6=12 Marks]**

60. [i] Realize the following expressions by using NAND gates only:

$g = (\bar{a} + \bar{b} + c)\bar{d}(\bar{a} + e)f$ where $\bar{x}$ represents the complement of x

[ii] Find the decimal equivalent of $(357.32)_8$ **[6+6=12 Marks]**

61. Develop an algorithm for Regula-Falsi method to find a root of $f(x) = 0$ starting with two initial iterates x_0 and x_1 to the root such that $\operatorname{sign}(f(x_0)) \neq \operatorname{sign}(f(x_1))$. Take n as the maximum number of iterations allowed and epsilon be the prescribed error. **[30 Marks]**

62. Using Lagrange interpolation formula, calculate the value of $f(3)$ from the following table of values of x and $f(x)$:

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

[15 Marks]

63. Find the value of $y(1.2)$ using Runge-Kutta fourth order method with step size $h = 0.2$ from the initial value problem: $y' = xy,\ y(1) = 2$ **[15 Marks]**

2008

64. Find the smallest positive root of equation $xe^x - \cos x = 0$ using Regula-Falsi method. Do three iterations. **[12 Marks]**

65. State the principle of duality

(i) in Boolean algebra and give the dual of the Boolean expressions $(X+Y).(\overline{X}.\overline{Z}).(Y+Z)$ and $X\overline{X} = 0$

(ii) Represent $(\overline{A}+\overline{B}+\overline{C})(A+\overline{B}+C)(A+B+\overline{C})$ in NOR to NOR logic network.

[6+6=12 Marks]

66. [i] The following values of the function $f(x) = \sin x + \cos x$ are given:

x	10^0	20^0	30^0
$f(x)$	1.1585	1.2817	1.3360

Construct the quadratic interpolating polynomial that fits the data. Hence calculate $f\left(\frac{\pi}{12}\right)$. Compare with exact value.

[ii] Apply Gauss-Seidel method to calculate x, y, z from the system:

$$-x - y + 6z = 42$$
$$6x - y - z = 11.33$$
$$-x + 6y - z = 32$$

with initial values $(4.67,\ 7.62,\ 9.05)$. Carry out computations for two iterations

[15+15=30 Marks]

67. Draw a flow chart for solving equation $F(x) = 0$ correct to five decimal places by Newton-Raphson method **[30 Marks]**

2007

68. Use the method of false position to find a real root of $x^3 - 5x - 7 = 0$ lying between 2 and 3 and correct to 3 places of decimals. **[12 Marks]**

69. Convert:
(i) 46655 given to be in the decimal system into one in base 6.
(ii) $(11110.01)_2$ into a number in the decimal system. **[6+6=12 Marks]**

70. [i] Find from the following table, the area bounded by the x-axis and the curve $y = f(x)$ between $x = 5.34$ and $x = 5.40$ using the trapezoidal rule:

x	5.34	5.35	5.36	5.37	5.38	5.39	5.40
$f(x)$	1.82	1.85	1.86	1.90	1.95	1.97	2.00

[15 Marks]

[ii] Apply the second order Runge-Kutta method to find an approximate value of y at $x = 0.2$ taking $h = 0.1$, given that y satisfies the differential equation and the initial condition $y' = x + y, y(0) = 1$ **[15 Marks]**

2006

71. Evaluate $I = \int_0^1 e^{-x^2} dx$ by the Simpson's rule

$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2)\right] + 4f(x_3) + ... + 2f(x_{2n-2}) + 4f(x_{2n-1}) + f(x_{2n})\right]$ with $2n = 10, \Delta x = 0.1, x_0 = 0, x_1 = 0.1, ..., x_{10} = 1.0$ **[12 Marks]**

72. [i] Given the number 59.625 in decimal system. Write its binary equivalent.
[ii] Given the number 3898 in decimal system. Write its equivalent in system base 8. **[6+6=12 Marks]**

73. If Q is a polynomial with simple roots $\alpha_1, \alpha_2, ...\alpha_n$ and if P is a polynomial of degree $< n$, show that $\frac{P(x)}{Q(x)} = \sum_{k=1}^{n} \frac{P(\alpha_k)}{Q'(\alpha_k)(x - \alpha_k)}$. Hence prove that there exists a unique polynomial of degree with given values c_k at the point α_k, $k = 1, 2, ...n.$. **[30 Marks]**

74. Draw a flowchart and algorithm for solving the following system of 3 linear equations in 3 unknowns x_1, x_2 & x_3 : $C * X = D$ with $C = \left(c_{ij}\right)^3_{i,j=1}, X = \left(x_j\right)^3_{j=1}, D = \left(d_i\right)^3_{i=1}$ **[30 Marks]**

2005

75. Use appropriate quadrature formulae out of the Trapezoidal and Simpson's rules to numerically integrate $\int_0^1 \frac{dx}{1 + x^2}$ with $h = 0.2$. Hence obtain an approximate value of π. Justify the use of particular quadrature formula. **[12 Marks]**

76. Find the hexadecimal equivalent of $(41819)_{10}$ and decimal equivalent of $(111011.10)_2$ **[12 Marks]**

77. Find the unique polynomial $P(x)$ of degree 2 or less such that $P(1) = 1, P(3) = 27, P(4) = 64$. Using the Lagrange's interpolation formula and the Newton's divided difference formula, evaluate $P(1.5)$ **[30 Marks]**

78. Draw a flow chart and also write algorithm to find one real root of the nonlinear equation $x = \phi(x)$ by the fixed-point iteration method. Illustrate it to find one real root, correct up to four places of decimals, of $x^3 - 2x - 5 = 0$. **[30 Marks]**

2004

79. The velocity of a particle at distance from a pint on it s path is given by the following table:

S(meters)	0	10	20	30	40	50	60
V(m/sec)	47	58	64	65	61	52	38

Estimate the time taken to travel the first 60 meters using Simpson's $\frac{1}{3}$ rd rule. Compare the result with Simpson's $\frac{3}{8}$ th rule. **[12 Marks]**

80. [i] If $(ABCD)_{16} = (x)_2 = (y)_8 = (z)_{10}$ then find $x, y \,\&\, z$

[ii] In a 4-bit representation, what is the value of 1111 in signed integer form, unsigned integer form, signed 1's complement form and signed 2's complement form? **[6+6=12 Marks]**

81. How many positive and negative roots of the equation $e^x - 5\sin x = 0$ exist? Find the smallest positive root correct to 3 decimals, using Newton-Raphson method. **[10 Marks]**

82. Using Gauss-Seidel iterative method, find the solution of the following system:

$$4x - y + 8z = 26$$
$$5x + 2y - z = 6$$
$$x - 10y + 2z = -13$$

up to three iterations. **[15 Marks]**

2003

83. Evaluate $\int_0^1 e^{-x^2} dx$ by employing three points Gaussian quadrature formula, finding the required weights and residues. Use five decimal places for computation. **[12 Marks]**

84. [i] Convert the following binary number into octal and hexa decimal system: 101110010.10010

[ii] Find the multiplication of the following binary numbers: 11001.1 and 101.1 **[6+6=12 Marks]**

85. Find the positive root of the equation $2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1}$ using Newton-Raphson method correct to four decimal places. Also show that the following scheme has error of second order:

$$x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2}\right)$$

[30 Marks]

86. Draw a flow chart and algorithm for Simpson's $\frac{1}{3}$ rd rule for integration $\int_a^b \frac{1}{1+x^2} dx$ correct to 10^{-6} **[30 Marks]**

2002

87. Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by the method of false position. **[12 Marks]**

88. [i] Convert $(100.85)_{10}$ into its binary equivalent.

[ii] Multiply the binary numbers $(1111.01)_2$ and $(1101.11)_2$ and check with its decimal equivalent **[4+8=12 Marks]**

89. [i] Find the cubic polynomial which takes the following values: $y(0)=1,\ y(1)=0,\ y(2)=1\ \&\ y(3)=10$. Hence, or otherwise, obtain $y(4)$

[ii] Given: $\frac{dy}{dx}=y-x$ where $y(0)=2,$ using the Runge-Kutta fourth order method, find $y(0.1)$ and $y(0.2)$. Compare the approximate solution with its exact solution. $\left(e^{0.1}=1.10517, e^{0.2}=1.2214\right)$.

[10+20=30 Marks]

90. Draw a flow chart to examine whether a given number is a prime. **[10 Marks]**

2001

91. Show that the truncation error associated with linear interpolation of $f(x)$, using ordinates at x_0 and x_1 with $x_0 \le x \le x_1$ is not larger in magnitude than $\frac{1}{8}M_2(x_1-x_0)^2$ where $M_2=\max\left|f''(x)\right|$ in $x_0 \le x \le x_1$. Hence show that if $f(x)=\frac{2}{\sqrt{\pi}}\int_0^{\pi} e^{-t^2}dt,$ the truncation error corresponding to linear interpolation of $f(x)$ in $x_0 \le x \le x_1$ cannot exceed $\frac{(x_1-x_0)^2}{2\sqrt{2\pi e}}$. **[12 Marks]**

92. [i] Given $A.B'+A'.B=C$ show that $A.C'+A'.C=B$

[ii] Express the area of the triangle having sides of lengths $6\sqrt{2},\ 12,\ 6\sqrt{2}$ units in binary number system. **[6+6=12 Marks]**

93. Using Gauss Seidel iterative method and the starting solution $x_1=x_2=x_3=0,$ determine the solution of the following system of equations in two iterations

$$10x_1-x_2-x_3=8$$
$$x_1+10x_2+x_3=12.$$
$$x_1-x_2+10x_3=10$$

Compare the approximate solution with the exact solution **[30 Marks]**

94. Find the values of the two-valued variables $A,B,C\ \&\ D$ by solving the set of simultaneous equations

$$A'+A.B=0$$
$$A.B=A.C$$
$$A.B+A.C'+C.D=C'.D$$

[15 Marks]

2000

95. [i] Using Newton-Raphson method, show that the iteration formula for finding the reciprocal of the p^{th} root of N is $x_{i+1}=\frac{x_i\left(p+1-Nx_i\right)}{p}$

[ii] Prove De Morgan's Theorem $(p+q)'=p'.q'$ **[6+6=12 Marks]**

96. [i] Evaluate $\int_0^1 \frac{dx}{1+x^2}$, by subdividing the interval $(0,\ 1)$ into 6 equal parts and using Simpson's one-third rule. Hence find the value of π and actual error, correct to five places of decimals

[ii] Solve the following system of linear equations, using Gauss-elimination method:

$$x_1 + 6x_2 + 3x_3 = 6$$
$$2x_1 + 3x_2 + 3x_3 = 117$$
$$4x_1 + x_2 + 2x_3 = 283$$

[15+15=30 Marks]

1999

97. Obtain the Simpson's rule for the integral $I = \int_a^b f(x)dx$ and show that this rule is exact for polynomials of degree $n \le 3$. In general show that the error of approximation for Simpson's rule is given by $R = -\frac{(b-a)^5}{2880} f^{iv}(\eta),\ \eta \in (0,2)$.

Apply this rule to the integral $\int_0^1 \frac{dx}{1+x}$ and show that $|R| \le 0.008333.$ **[20 Marks]**

98. Using fourth order classical Runge-Kutta method for the initial value problem $\frac{du}{dt} = -2tu^2, u(0) = 1,$ with $h = 0.2$ on the interval $[0,\ 1]$, calculate $u(0.4)$ correct to six places of decimal. **[20 Marks]**

1998

99. Evaluate $\int_1^3 \frac{dx}{x}$ by Simpson's rule with 4 strips. Determine the error by direct integration. **[20 Marks]**

100. By the fourth –order Runge-Kutta method. tabulate the solution of the differential equation $\frac{dy}{dx} = \frac{xy+1}{10y^2+4},\ y(0) = 0$ in $[0,\ 0.4]$ with step length 0.1 correct to five places of decimals

[20 Marks]

101. Use Regula-Falsi method to show that the real root of $x\log_{10} x - 1.2 = 0$ lies between 3 and 2.740646 **[20 Marks]**

1997

102. Apply that fourth order Runge-Kutta method to find a value of y correct to four places of decimals at $x = 0.2$, when $y' = \frac{dy}{dx} = x + y,\ y(0) = 1$ **[20 Marks]**

103. Show that the iteration formula for finding the reciprocal of N is $x_{n+1} = x_n\left(2 - N_{xn}\right),\ n = 0,\ 1...$

[20 Marks]

104. Obtain the cubic spline approximation for the function given in the tabular form below:

x	0	1	2	3
$f(x)$	1	2	33	244

and $M_0 = 0, M_3 = 0$ **[20 Marks]**

1996

105. Describe Newton-Raphson method for finding the solutions of the equation $f(x) = 0$ and show that the method has a quadratic convergence. **[20 Marks]**

106. The following are the measurements t made on a curve recorded by the oscillograph representing a change of current i due to a change in the conditions of an electric current:

t	1.2	2.0	2.5	3.0
i	1.36	0.58	0.34	0.20

Applying an appropriate formula interpolate for the value of i when $t = 1.6$ **[20 Marks]**

107. Solve the system of differential equations $\frac{dy}{dx} = xz + 1,\ \frac{dz}{dx} = -xy$ for $x = 0.3$ given that $y = 0$ and $z = 1$ when $x = 0,$ using Runge-Kutta method of order four **[20 Marks]**

1995

108. Find the positive root of $\log_e x = \cos x$ nearest to five places of decimal by Newton-Raphson method. **[20 Marks]**

109. Find the value of $\int_{1.6}^{3.4} f(x)\, dx$ from the following data using Simpson's $\frac{3}{8}$ rd rule for the interval $(1.6,\ 2.2)$ and $\frac{1}{8}$ th rule for $(2.2,\ 3.4)$:

x	1.6	1.8	2.0	2.2	2.4
$f(x)$	4.953	6.050	7.389	9.025	11.023

x	2.6	2.8	3.0	3.2	3.4
$f(x)$	13.464	16.445	20.086	24.533	29.964

[20 Marks]

1994

110. Find the positive root of the equation $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} e^{0.3x}$ correct to five decimal places. **[20 Marks]**

111. Fit the following four points by the cubic splines.

i	0	1	2	3
x_i	1	2	3	4
y_i	1	5	11	8

Use the end conditions Use the end conditions $y''_0 = y''_3 = 0$

Hence compute **[i]** $y(1.5)$

[ii] $y'(2)$ **[20 Marks]**

112. Find the derivative of $f(x)$ at $x = 0.4$ from the following table:

x	0.1	0.2	0.3	0.4
$y = f(x)$	1.10517	1.22140	1.34986	1.49182

[20 Marks]

1993

113. Find correct to 3 decimal places the two positive roots of $2e^x - 3x^2 = 2.5644$ **[20 Marks]**

114. Evaluate approximately $\int_{-3}^{3} x^4 \, dx$ Simpson's rule by taking seven equidistant ordinates. Compare it with the value obtained by using the trapezoidal rule and with exact value. **[20 Marks]**

115. Solve $\frac{dy}{dx} = xy$ for $x = 1.4$ by Runge-Kutta method, initially $x = 1, y = 2$ (Take $h = 0.2$) **[20 Marks]**

1992

116. Compute to 4 decimal placed by using Newton-Raphson method, the real root of $x^2 + 4\sin x = 0$. **[20 Marks]**

117. Solve by Runge-Kutta method $\frac{dy}{dx} = x + y$ with the initial conditions $x_0 = 0, y_0 = 1$ correct up to 4 decimal places, by evaluating up to second increment of y (Take $h = 0.1$) **[20 Marks]**

118. Fit the natural cubic spline for the data. **[20 Marks]**

$x:$	0	1	2	3	4
$y:$	0	0	1	0	0