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UPSC MATHS
OPTIONAL COACHING

Linear Programming

Previous year Questions from 2020 to 1992

2021-22

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2020

1. UPSC maintenance section has purchased sufficient number of curtain cloth pieces to meet the curtain requirement of its building. The length of each piece is 17 feet. The requirement according to curtain length is as follows:

Curtain length (in feet)	Number required
5	700
9	400
7	300

The width of all curtains is same as that of available pieces. Form a linear programming problem in standard from that decided the number of pieces cut in different ways so that the total trim loss is minimum. Also give a basic feasible solution to it. **[10 Marks]**

2. Solve the linear programming problem using simple method:

Minimize $z = -6x_1 - 2x_2 - 5x_3$

subject to $2x_1 - 3x_2 + x_3 \le 14$

$-4x_1 + 4x_2 + 10x_3 \le 46$

$2x_1 + 2x_2 - 4x_3 \le 37$

and $x_1 \ge 2, x_2 \ge 1, x_3 \ge 3$ **[15 Marks]**

3. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method and use it to find the optimal solution and the transportation cost of the problem. **[20 Marks]**

		Destinations				
		D_1	D_2	D_3	D_4	Demand
Sources	S_1	10	0	20	11	15
	S_2	12	8	9	20	25
	S_3	0	14	16	18	10
	Supply	5	20	15	10	

2019

4. Use graphical method to solve the linear programming problem.

Maximize $z = 3x_1 + 2x_2$

subject to

$x_1 - x_2 \ge 1$

$x_1 + x_3 \ge 3$

and $x_1, x_2, x_3 \ge 0$ **[10 Marks]**

5. Solve the linear programming problem using Simplex Method.

Minimize $Z = x_1 + 2x_2 - 3x_3 - 2x_4$

subject to

$x_1 + 2x_2 - 3x_3 + x_4 = 4$

$x_1 + 2x_2 - x_3 + 2x_4 = 4$

and $x_1, x_2, x_3, x_4 \ge 0$ **[15 Marks]**

6. Consider the following LPP,

Maximize $z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

subject to

$x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$

and $x_1, x_2, x_3, x_4 \geq 0$

Use the dual problems to verify that the basic solution (x_1, x_2) is not optimal **[10 Marks]**

2018

7. An agricultural firm has 180 tons of nitrogen fertilizer, 250 tons of phosphate and 220 tons of potash. It will be able to sell a mixture of these substances in their respective ratio 3: 3: 4 at a profit of Rs.1500 Per ton and a mixture in the ratio 2: 4: 2 at a profit of Rs. 1200 per ton. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. **[10 Marks]**

8. Solve the following liner programming problem by Big M-method and show that the problem has finite optional has finite optimal solutions. Also Find the value of the objective Function: **[20 Marks]**

Minimize $z = 3x_1 + 5x_2$

Subject to
$$x_1 + 2x_2 \geq 8$$
$$3x_1 + 2x_2 \geq 12$$
$$5x_1 + 6x_2 \leq 60,$$
$$x_1, x_2 \geq 0.$$

9. In a factory there are five operators $O_1, O_2, O_3, O_4, O_5,$ and five machines M_1, M_2, M_3, M_4, M_5 The operating costs are given when the O_i operator. Operates the M_j machine $(i, j = 1, 2, ..., 5)$. .But there is a restriction that O_3 cannot be allowed to operate the third machine M_3 and O_2 cannot be allowed to operate the fifth machine M_5 The cost matrix is given below. Find the optimal assignment and the optimal assignment cost also. **[20 Marks]**

		Machines				
		M_1	M_2	M_3	M_4	M_5
Operator	O_1	24	29	18	32	19
	O_2	17	26	34	22	21
	O_3	27	16	28	17	25
	O_4	22	18	28	30	24
	O_5	28	16	31	24	27

10. How many basic solutions are there in the following linearly independent set of equations? Find all of them. **[15 Marks]**

$2x_1 - x_2 + 3x_3 + x_4 = 6$

$4x_1 - 2x_2 - x_3 + 2x_4 = 10$

2017

11. Using graphical method, find the maximum values of $2x + y$

Subject to

$4x + 3y \leq 12$

$4x + y \leq 8$

$4x - y \leq 8$

$x, y \geq 0$

[10 Marks]

12. Solve the following linear programming problem by simplex method:

Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to

$$2x_1 + 3x_2 \le 8$$
$$2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15x_3$$
and $x_1, x_2, x_3 \ge 0.$

[20 Marks]

13. Find the initial basic feasible solution of the following transportation problem using Vogel's approximation methods and find the cost.

		Destinations					
		D_1	D_2	D_3	D_4	D_5	
Origins	O_1	4	7	0	3	6	14
	O_2	1	2	−3	3	8	9
	O_3	3	−1	4	0	5	17
		8	3	8	13	8	

Supply

Demand

[15 Marks]

2016

14. Find the maximum value of $5x + 2y$ with constraints $x + 2y \ge 1, 2x + y \le 1, x \ge 0$ by $y \ge 0$ graphically method. **[10 marks]**

15. Maximize $z = 2x_1 + 3x_2 + 6x_3$

Subjected to

$2x_1 + x_2 + x_3 \le 5$

$3x_2 + 2x_3 + \le 6$

and $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$

Is the optimal solution unique? Justify your answer. **[20 marks]**

2015

16. Solve the following assignment problem to *maximize* the sales

		Territoreis				
		I	*II*	*III*	*IV*	*V*
Salesmen	*A*	3	4	5	6	7
	B	4	15	13	7	6
	C	6	13	12	5	11
	D	7	12	15	8	5
	E	8	13	10	6	9

[10 Marks]

17. Consider the following linear programming problem

Maximize $Z = x_1 + 2x_2 - 3x_3 + 4x_4$

subject to

$$x_1 + x_2 + 2x_3 + 3x_4 = 12$$
$$x_2 + 2x_3 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \ge 0$$

[20 Marks]

(i) Using the definition, find its all *basic solutions*. Which of these are *degenerate basic feasible solutions* and which *are non-degenerate basic feasible solutions*?

(ii) Without solving the problem, show that it has an optimal solution and which of the *basic feasible solution(s)* is/are *optimal*?

18. Solve the following linear programming problem by the *simplex method*. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem:

Maximize $Z = 2x_1 - 4x_2 + 5x_3$

subject to

$$x_1 + 4x_2 - 2x_3 \le 2$$
$$-x_1 + 2x_2 + 3x_3 \le 1$$
$$x_1, x_2, x_3 \ge 0$$

[20 Marks]

2014

19. Solve graphically:

Maximize $Z = 6x_1 + 5x_2$

Subjet to

$$2x_1 + x_2 \le 16$$
$$x_1 + x_2 \le 11$$
$$x_1 + 2x_2 \ge 6$$
$$5x_1 + 6x_2 \le 90$$

[10 Marks]

20. Find the initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also, find its optimal solution and the minimum transportation cost

		D_1	D_2	D_3	D_4	Supply
Origins	O_1	6	4	1	5	14
	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
Demand		6	10	15	4	

[20 Marks]

21. Find all *optimal solutions* of the following linear programming problem by the *simplex method*:

Maximize $Z = 30x_1 + 24x_2$

Subject to

$$5x_1 + 4x_2 \le 200$$
$$x_1 \le 32$$
$$x_2 \le 40$$
$$x_1 x_2 \ge 0$$

[20 Marks]

2013

22. Solve the following linear programming problem

Maximize $Z = 2x_1 + 3x_2 - 5x_3$

Subject to

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \ge 10$$
$$x_1, x_2, x_3 \ge 0$$

[10 Marks]

23. Solve the minimum time assignment problem

	Machines			
	M_1	M_2	M_3	M_4
Jobs J_1	3	12	5	14
Jobs J_2	7	9	8	12
Jobs J_3	5	11	10	12
Jobs J_4	6	14	4	11

[15 Marks]

24. Solve the following linear programming problem

Minimize $Z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

Subjet to the constraints

$$x_1 + 2x_2 - 2x_3 + 4x_4 \le 40$$
$$2x_1 - x_2 + x_3 + 2x_4 \le 8$$
$$4x_1 - 2x_2 + x_3 - x_4 \le 10$$
$$\text{and } x_1, x_2, x_3, x_4 \ge 0$$

[20 Marks]

2012

25. For each hour per day that Ashok studies mathematics, it yields him 10 marks and for each hour that he studies physics, it yields him 5 marks. He can study at most 14 hours a day and he must get at least 40 marks in each. Determine graphically how many hours a day he should study mathematics and physics each, in order to maximize his marks? **[12 Marks]**

26. By the method of Vogel, determine an initial basic feasible solution for the following transportation problem: Products $P_1, P_2, P_3, \& P_4$ have to be sent of destinations $D_1, D_2 \& D_3$. The cost of sending product P_i to destinations D_j is C_{ij}, where the matrix

$$\left[C_{ij}\right] = \begin{bmatrix} 10 & 0 & 15 & 5 \\ 7 & 3 & 6 & 15 \\ 0 & 11 & 9 & 13 \end{bmatrix}$$

[12 Marks]

The total requirements of destinations $D_1, D_2 \& D_3$ are given by 45, 45, 95 respectively and the availability of the products $P_1, P_2, P_3, \& P_4$ are respectively 25, 35, 55 and 70.

2011

27. Solve by simplex method, the following LP Problem:

Maximize, $Z = 5x_1 + x_2$

Subject to constraints,

$$3x_1 + 5x_2 \le 15$$
$$5x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0$$

[12 Marks]

28. Write down the dual of the following LP problem and hence solve it by graphical method:

Minimize $Z = 6x_1 + 4x_2$

Subject to constraints

$$2x_1 + x_2 \geq 1$$
$$3x_1 + 4x_2 \geq 1.5$$
$$x_1, x_2 \geq 0$$

[20 Marks]

2010

29. Construct the dual of the primal problem:

Maximize $Z = 2x_1 + x_2 + x_3$

Subject to the constraints

$$x_1 + x_2 + x_3 \geq 6$$
$$3x_1 - 2x_2 + 3x_3 = 3$$
$$-4x_1 + 3x_2 - 6x_3 = 1$$
$$x_1, x_2, x_3 \geq 0$$

[12 Marks]

30. Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories to five warehouses W_1, W_2, W_3, W_4, W_5 is minimized. The five warehouses must receive 40 tons, 50 tons, 70 tons, .90 tons and 90 tons respectively. The availability of the material at F_1, F_2, F_3 is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below:

	W_1	W_2	W_3	W_4	W_5
F_1	4	1	2	6	9
F_2	6	4	3	5	7
F_3	5	2	6	4	8

Use Vogel's approximation method to obtain the initial basic feasible solution. **[30 Marks]**

2009

31. A paint factory produces both interior and exterior paint from two raw materials M_1 and M_2. The basic data is as follows:

	Tons of raw material per ton of		**Maximum daily availability**
	Exterior paint	**Interior paint**	
Raw Material M_1	6	4	24
Raw Material M_2	1	2	6
Profit per ton (Rs. 1000)	5	4	

A market survey indicates that the daily demand interior paint cannot exceed that of exterior paint by more than 1 ton. The maximum daily demand of interior paint is 2 tons. The factory wants to determine the optimum product mix of interior and exterior paint that maximizes daily profits. Formulate the LP problem for this situation **[12 Marks]**

32. Solve the following linear programming problem:

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subjet to

$$2x_1 + 3x_2 \le 8$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$
$$2x_2 + 5x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

[30 Marks]

2008

33. Find the dual of the following linear programming problem: **[12 Marks]**

Max $Z = 2x_1 - x_2 + x_3$

Subjet to

$$x_1 + x_2 - 3x_3 \le 8$$
$$4x_1 - x_2 + x_3 = 2$$
$$2x_1 + 3x_2 - x_3 \ge 5$$
$$x_1, x_2, x_3 \ge 0$$

34. Solve the following transportation problem: **[30 Marks]**

Destinations

		D_1	D_2	D_3	D_4	D_5	D_6	Availability
Factories	F_1	2	1	3	3	2	5	50
	F_2	3	2	2	4	3	4	40
	F_3	3	5	4	2	4	1	60
	F_4	4	2	2	1	2	2	30
	Demand	30	50	20	40	30	10	180

by finding the initial solution by Matrix Minima Method

2007

35. Put the following in slack form and describe which of the variables are 0 at each of the vertices of the constraint set and hence determine the vertices algebraically:

Maximize $Z = 4x + 3y$

Subjet to

$$x + y \le 4$$
$$-x + y \le 2$$
$$x, y \ge 0$$

[12 Marks]

36. Solve the following by Simplex method:

Maximize $u = 4x + 3y$

Subjet to

$$x + y \le 1$$
$$x - 2y \le 4$$
$$x, y \ge 0$$

[30 Marks]

2006

37. Given the programme **[12 Marks]**

Maximize $u = 5x + 2y$

Subjet to

$$x + 3y \le 12$$
$$3x - 4y \le 9$$
$$7x + 8y \le 20$$
$$x, y \ge 0$$

38. Use the simplex method to solve the problem **[30 Marks]**

Maximize $u = 2x + 3y$

Subjet to

$$-2x + 3y \le 2$$
$$3x + 2y \le 5$$
$$x, y \ge 0$$

2005

39. Put the following program in standard form: **[12 Marks]**

Minimize $Z = 25x_1 + 30x_2$

Subjet to

$$4x_1 + 7x_2 \ge 1$$
$$8x_1 + 5x_2 \ge 3$$
$$6x_1 + 9x_2 \ge -2$$
$$x_1, x_2 \ge 0$$

40. Use the simplex method to solve the problem **[30 Marks]**

Maximize $Z = 5x_1 + 2x_2$

Subjet to

$$6x_1 + x_2 \ge 6$$
$$4x_1 + 3x_2 \ge 12$$
$$x_1 + 2x_2 \ge 4$$

and $x_1, x_2 \ge 0$

2004

41. Use simplex method to solve the linear programming problem: **[12 Marks]**

Max $Z = 3x_1 + 2x_2$

Subjet to

$$x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 2$$
$$x_1, x_2 \ge 0$$

42. A travelling salesman has to visit 5 cities. He wishes to start from a particular city, visit each city once and then return to his starting point. Cost of going from one city to another is given below:

	A	B	C	D	E
A	∞	4	10	14	2
B	12	∞	6	10	4
C	16	14	∞	8	14
D	24	8	12	∞	10
E	2	6	4	16	∞

You are required to find the least cost route. **[15 Marks]**

43. A department has 4 technicians and 4 tasks are to be performed. The technicians differ in efficiency and tasks differ in their intrinsic difficulty. The estimate of time (in hours), each technician would take to perform a task is given below. How should the task be allotted, one to a technician, so as to minimize the total work hours? **[15 Marks]**

Task / Technician	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

2003

44. For the following system of equations **[12 Marks]**

$x_1 + x_2 + x_3 = 3$

$2x_1 - x_2 + 3x_3 = 4$

Determine:

(i) All basic solutions

(ii) All basic feasible solutions

(iii) A feasible solution which is not a basic feasible solution.

45. An animal feed company must produce 200 kg of a mixture consisting of ingredients X_1 and X_2 daily. X_1 costs Rs. 3 per Kg and X_2 costs Rs. 8 per Kg. No more than 80 Kg of X_1 can be used, and at least 60 Kg of X_2 must be used. Formulate a linear programming model of the problem and use Simplex method to determine the ingredients X_1 and X_2 to be used to minimize cost. **[15 Marks]**

46. Find the optimal solution for the assignment problem with the following cost matrix: **[15 Marks]**

$$\begin{bmatrix} 6 & 1 & 9 & 11 & 12 \\ 2 & 8 & 17 & 2 & 5 \\ 11 & 8 & 3 & 3 & 3 \\ 4 & 10 & 8 & 6 & 11 \\ 8 & 10 & 11 & 5 & 13 \end{bmatrix}$$

Indicate clearly the rule you apply to arrive at the complete assignment.

2002

47. Using Simplex method **[12 Marks]**

Maximize $Z = 45x_1 + 80x_2$

Subjet to

$$5x_1 + 20x_2 \le 400$$
$$10x_1 + 15x_2 \le 450$$
$$x_1, x_2 \ge 0$$

48. Using simplex method maximize **[15 Marks]**

Maximize $Z = 5x_1 + 3x_2$

Subjet to

$$x_1 + x_2 \le 2$$
$$5x_1 + 2x_2 \le 10$$
$$3x_1 + 8x_2 \le 12$$
$$x_1, x_2 \ge 0$$

49. A company has 3 factories A, B and C which supply units to warehouses X, Y and Z. Every month the capacities of the factories per month are 60, 70 and 80 units A, B and C respectively. The requirements of X, Y and Z are 50, 80 and 80 respectively. The necessary data in terms of unit transportation cost in rupees, factory capacities and warehouse requirements are given below: **[15 Marks]**

	X	Y	Z	
A	8	7	5	60
B	6	8	9	70
C	9	6	5	80
	50	80	80	210

Find the minimum distribution cost.

2001

50. Compute all basic feasible solutions of the linear programming problem **[12 Marks]**

Maximize $Z = 2x_1 + 3x_2 + 2x_3$

Subjet to

$$2x_1 + 3x_2 - x_3 = 8$$
$$x_1 - 2x_2 + 6x_3 = -3$$
$$x_1, x_2, x_3 \ge 0$$

And hence indicate the optimal solution.

51. Using duality or otherwise solve the linear programming problem **[12 Marks]**

Minimize $Z = 18x_1 + 12x_2$

Subjet to

$$2x_1 - 2x_2 \ge -3$$
$$3x_1 + 2x_2 \ge 3$$
$$x_1, x_2 \ge 0$$

52. A manufacturer has distribution centers at Delhi, Kolkata and Chennai. These centers have available 30, 50 and 70 units of his product. His four retail outlets require the following number of units: A, 30: B, 20; C, 60; D, 40. The transportation cost per unit in rupees between each center and outlet is given in the following table:

Distribution Centers	Retail outlets			
	A	B	C	D
Delhi	10	7	3	6
Kolkata	1	6	7	3
Chennai	7	4	5	3

Determine the minimum transportation cost. **[20 Marks]**

2000

53. An explosion in a factory manufacturing explosive can occur because of
(i) Leakage of electricity,
(ii) Defects in machinery,
(iii) Carelessness of workers or
(iv) Sabotage.
The probability that there is a leakage of electricity is 0.20, the machinery is defective is 0.30, the workers are careless is 0.40, there is sabotage is 0.10. The engineers feel that an explosion can occur with probability
(i) 0.25 because of leakage of electricity,
(ii) 0.20 because of defects in machinery,
(iii) 0.50 because of carelessness of workers and
(iv) 0.75 because of sabotage.
Determine the most likely cause of explosion. **[20 Marks]**

54. Two unbiased coins are tossed once (independently) and the number X of heads that turned up is noted. A number is selected at random from $X, X+1$ and $X+2$. If Y is the number selected, find the joint distribution of X and Y. Also obtain the expectation of XY. **[20 Marks]**

55. Solve the following assignment problem for the given assignment costs: **[20 Marks]**

Job \ Person	*I*	*II*	*III*	*IV*	*V*
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	13

1999

56. A police department has the following minimal daily requirements form police officers during its six shift periods: -

Time of Day	Period	Minimal Number Required
2 a.m.-6 a.m.	1	22
6 a.m.-10 a.m.	2	55
10 a.m.-2 p.m.	3	88
2 p.m.-6 p.m.	4	110
6 p.m.-10 p.m.	5	44
10 p.m.-2 a.m.	6	33

An officer must start at the beginning of a 4-hour shift and stay on duty for two consecutive shifts (an 8-hour tour). Any one starting during period 6 stays on duty during period 1 of the next day. The objective of the police department is to always have on duty the minimal number required in a period but to do so with the least number of officers. Develop the corresponding linear programming model. **[20 Marks]**

57. Show that a problem in the theory of games can be expressed as a linear programming problem. **[20 Marks]**

58. Respond True or False to the following, justify your answer in case of False:
   (i) If the number of primal variables is much smaller than the number of constraints, it is more efficient to obtain the solution of the primal by solving its dual.
   (ii) When the primal problem is non-optimal, the dual problem is automatically infeasible.
   (iii) An unrestricted primal variable will have the effect of yielding an equality dual constraint.
   (iv) If the solution space is unbounded, the objective value always will be unbounded.
   (v) The selection of the entering variable from among the current non-basic variable as the one with the most negative objective coefficient guarantees the most increase in the objective value in the next iteration.
   (vi) In the simplex method, the feasibility conditions for the maximization and minimization problems are different.
   (vii) A simplex iteration (basic solution) may not necessarily coincide with a feasible extreme point of the solution space.
   (viii) If the leaving variable does not correspond to the minimum ratio, at least one basic variable will definitely become negative in the next iteration. **[20 Marks]**

59. Develop mathematical model of a balanced transportation problem. Prove that it always has a feasible solution. **[20 Marks]**

60. Find the optimal assignment for the given assignment costs:

		Machine		
		1	2	3
Job	1	5	7	9
	2	14	10	12
	3	15	13	16

[20 Marks]

61. Give the economic interpretation of duality in linear programming. **[20 Marks]**

1998

62. Prove that a basic feasible solution to a linear programming problem must correspond to an extreme point of the set of all feasible solutions. **[20 Marks]**

63. Solve the unbalanced assignment problem in minimization where

$$[C_{ij}] = \begin{bmatrix} 12 & 10 & 15 & 22 & 18 & 8 \\ 10 & 18 & 25 & 15 & 16 & 12 \\ 11 & 10 & 3 & 8 & 5 & 9 \\ 6 & 14 & 10 & 13 & 13 & 12 \\ 8 & 12 & 11 & 7 & 13 & 10 \end{bmatrix}$$ **[20 Marks]**

64. A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles depositors only. It has been found that the service time distributions of both deposits and withdrawals are exponential with a mean service time of 3 minutes per customer. Depositors and withdrawers are found to arrive in a Poisson fashion throughout the day with mean arrival rate of 16 and 14 per hour. What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both the withdrawals and deposits? What would be the effect if this could only be accomplished by increasing the service time to 3.5 minutes? **[20 Marks]**

65. A bookbinder processes the manuscripts of five books through the three stages of operation, viz., printing, binding and finishing. The time required to perform the printing, binding and finishing operations are given below:

Book	Processing Time (in hours)		
	Printing	Binding	Finishing
1	50	60	90
2	100	70	110
3	90	30	70
4	70	40	80
5	60	50	110

Determine the order in which books should be processed in order to minimize the total time required to process the books. Find the minimum total processing time. **[20 Marks]**

1997

66. State the Transportation problem in general terms and explain the problem of degeneracy **[20 Marks]**

67. Use simplex method to solve the following Linear Programming Problem:

Maximize $Z = 4x_1 + 10x_2$

Subjet to

$$2x_1 + x_2 \le 50$$
$$2x_1 + 5x_2 \le 100$$
$$2x_1 + 3x_2 \le 90$$
$$x_1, x_2 \ge 0$$

[20 Marks]

68. In factory, there are six jobs to perform and each should go through two machines A and B in the order A, B. The processing timings (in hours) for the jobs are given below. Determine the sequence form performing the jobs that would minimize the total elapsed time T. What is the value of T?

		Jobs					
		J_1	J_2	J_3	J_4	J_5	J_6
Machines	A	1	3	8	5	6	3
	B	5	6	3	2	2	10

[20 Marks]

69. Solve that linear programming problem:

Maximize $Z = 3x_1 + 5x_2$

Subjet to

$$x_1 \le 4$$
$$x_2 \le 6$$
$$3x_1 + 2x_2 \le 18$$
$$x_1, x_2 \le 0$$

If the cost coefficient of x_1 is kept fixed, find the range for the cost coefficient of x_2 without affecting the optimal solution. **[20 Marks]**

70. A tax consulting firm has four service stations (counters) in its office to receive people who have problems and complaints about their income, wealth etc. The number of arrivals averages 80 persons in an eight-hour service day. Each tax adviser spends an irregular amount average service time is 20 minutes. Calculate the average number of people waiting to be serviced, average time a person spends in the system and the average waiting time for a person. What is the expected number of idle tax adviser at any specified time? **[20 Marks]**

71. Solve the assignment problem represented by the following for minimization of costs. Find also alternate solutions if any.

	I	*II*	*III*	*IV*	*V*	*VI*
A	11	24	60	13	21	29
B	45	80	74	52	65	50
C	43	30	93	39	47	35
D	76	44	29	51	41	34
E	38	13	59	24	27	27
F	5	58	55	33	19	30

[20 Marks]

72. A company has four plants P_1, P_2, P_3, P_4 from which it supplies to three markets M_1, M_2, M_3. Determine the optimal transportation plan using MODI method from the following data giving the plant to market shifting costs, quantities available at each plant and quantities required at each market: **[20 Marks]**

Market	**Plants**				**Required at**
	P_1	P_2	P_3	P_4	
M_1	21	16	25	13	11
M_2	17	18	14	23	13
M_3	32	27	18	41	19
Available at plant	6	10	12	15	43

73. Determine the

Maximum $Z = P_1 P_2 ... P_n$

subject to the constraints **[20 Marks]**

$$\sum_{i=1}^{n} c_i p_i \le x,\ 0 \le p_i \le 1\ (i = 1, 2, ...n)$$

(assume that $c_i > x\ \forall\ i$)

74. Determine the optimal sequence of jobs that minimizes the total elapsed time required to complete the following jobs and find the total elapsed time. The jobs are to be processed on three machines M_1, M_2, M_3 in the same order M_1, M_2, M_3 and processing times are as below:

Job

Machines	A	B	C	D	E	F	G
M_1	3	8	7	4	9	8	7
M_2	4	3	2	5	1	4	3
M_3	6	7	5	11	5	6	12

[20 Marks]

Find also the idle times for the three machines.

1995

75. Solve the following linear programming problem:

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

Subjet to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$
$$x_1, x_2, x_3, x_4 \geq 0$$

[20 Marks]

76. Solve the transportation problem below for minimizing the cost:

Costs	**Store**						**Availability**
	1	2	3	4	5	6	
1	9	12	9	6	9	10	5
2	7	3	7	7	5	5	6
3	6	5	9	11	3	11	2
4	6	8	11	2	2	10	9
Requirement	4	4	6	4	6	2	22

[20 Marks]

77. There are five jobs each of which must go through two machines A and B in the order A, B. Processing times are given below:

Job	**1**	**2**	**3**	**4**	**5**
Time for A(in hours)	7	3	11	5	12
Time for B(in hours)	4	8	9	10	6

Determine a sequence for the jobs that will minimize the elapsed time. Compute the total idle times for the machines in this period **[20 Marks]**

1994

78. Solve by using simplex method

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subjet to

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_2 \leq 460$$
$$x_1 + 4x_2 \leq 420$$
$$x_1, x_2, x_3 \geq 0$$

[20 Marks]

79. Consider the following data:

		Destinations			
		1	2	3	Capacities
Sources	1	2	2	3	10
	2	4	1	2	15
	3	1	3	x	40
	Demands	20	15	30	

The cost of shipment from third source to the third destination is not known. How many units should be transported from the sources to the destinations so that the total cost of transporting all the units to their destinations is a minimum? **[20 Marks]**

1993

80. Use simplex method to solve:

Maximize $x_0 = x_1 - 3x_2 + 2x_3$

Subjet to

$$3x_1 - x_2 + 2x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2, x_3 \le 0$$

[20 Marks]

81. A Departmental Head has four subordinates and four tasks are to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimates of the times each man would take to perform each task is given in the effectiveness matrix below. How should the tasks be allocated one to one man, so as to minimize the total man hours?

		Man			
		I	*II*	*III*	*IV*
Task	*A*	8	26	17	11
	B	13	28	14	26
	C	38	19	18	15
	D	19	26	24	10

[20 Marks]

1992

82. Solve the following linear programming problem

Maximize $Z = 3x_1 + 2x_2$

Subjet to

$$x_1 + x_2 \le 7$$
$$x_1 - x_2 \le 2$$
$$x_1, x_2 \ge 0$$

[20 Marks]

83. The following table gives the cost for transporting material from supply points A, B, C, D to demand points E, F, G, H, J:

		To				
		E	F	G	H	J
From	A	8	10	12	17	15
	B	15	13	18	11	9
	C	14	20	6	10	13
	D	13	19	7	5	12

The present allocation is as follows:

A to E 90; A to F 10; B to F 150; C to F 10

C to G 50; C to J 120; D to H 210; D to J 70

(i) Check if this allocation is optimum. If not, find an optimum schedule.

(ii) If in the above problem the transportation cost from A to G is reduce to 10, what will be the new optimum schedule? **[20 Marks]**